Triple Double

Problem: Published as a challenge to students and teachers in the February 2013 issue of *The Physics Teacher*.

You have two particles, one with charge q and mass m and another with charge $-q$ and mass $2m$. Initially separated a distance d and moving with velocities of equal magnitude, but one directed towards the other particle and one perpendicular to the line that connects the two particles.

Later in their motion (we can consider the electric force only) the particles reach a distance $3d$ twice. With this information we want to know the possible values of q.

Solution: Of the possible conical sections, the trajectory in the center of mass frame needs to be an ellipse because that is the only way in which the distance can be $3d$ twice. The total energy of the particles will be

$$E = -\frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{2a} = \frac{1}{2}\mu v_r{}^2 - \frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{r}$$

For the particles to reach a distance $3d$ the distance between the focal point and the edge of the ellipse needs to be at least 3d, so geometrically we need to satisfy the relation

$$\frac{\sqrt{d^2+(2a-d)^2}}{2} + a > 3d$$

Which means that $a > \frac{17d}{10}$

This condition indicates that $E = -\frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{2a} = \frac{1}{2}\mu v_r{}^2 - \frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{r} > -\frac{1}{4\pi\varepsilon_\circ}\frac{10q^2}{2\times 17d}$

At the starting point $\frac{1}{2}\frac{2m}{3}2v^2 - \frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{d} > -\frac{1}{4\pi\varepsilon_\circ}\frac{10q^2}{2\times 17d}$, so

$$v\sqrt{4\pi\varepsilon_\circ dm\left(\frac{17}{18}\right)} > q$$

The condition to be an ellipse is that $E = -\frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{2a} = \frac{1}{2}\mu v_r{}^2 - \frac{1}{4\pi\varepsilon_\circ}\frac{q^2}{r} < 0$, so

$$v\sqrt{4\pi\varepsilon_\circ dm\left(\frac{2}{3}\right)} < q$$