Know strings attached

Problem: Published as a challenge to students and teachers in the March 2017 issue of *The Physics Teacher*.

We have three small charges q and masses m, 4m and m. The balls are connected by light non-conducting strings of length *d* each and move without friction on a horizontal plane. Initially, the balls are at rest and form a straight line. Then, a quick horizontal kick gives the central ball a speed *v* perpendicular to the strings connecting the balls. Find the minimum subsequent distance between the balls of mass m.

Solution: In the initial condition, the kinetic energy of the system is given solely by the mass in the middle:

$$KE = \frac{1}{2}(4m)v^2 = 2mv^2$$

Similarly, initially the linear momentum is given solely by the mass in the middle:

$$P = 4mv$$

After that initial condition, the motion of the three charges will follow a cyclic pattern where the center of mass will move at constant velocity in the direction of v and the three masses will oscillate in a symmetric way, similar to a CO_2 molecule. This is due to the electric repulsion between them and the strings that constrain the distance between the middle mass and the ones on the sides.

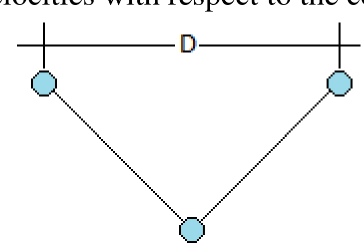
Conservation of momentum gives us the center of mass velocity

$$v_{CM} = \frac{4mv}{6m} = \frac{2}{3}v$$

And the kinetic energy associated with this motion of the center of mass

$$KE_{CM} = \frac{1}{2}(6m)\left(\frac{2v}{3}\right)^2 = \frac{4m}{3}v^2$$

The minimum distance between the side charges will occur when the oscillation reverses direction at which point the velocities with respect to the center of mass will be zero.



Then the difference in kinetic energy is traded for difference in electric potential energy.

$$2mv^2 - \frac{4m}{3}v^2 = \frac{q^2}{4\pi\varepsilon_o}\left(\frac{1}{D} - \frac{1}{2d}\right)$$

Solving for D we get:

$$D = \frac{2d}{1 + \frac{16m\pi\varepsilon_o dv^2}{3q^2}}$$