Impulse control

Problem: Published as a challenge to students and teachers in the May 2018 issue of *The Physics Teacher*.

A charged particle enters a region with a viscous fluid, where the retarding force is proportional to the velocity and it stops 20cm from the entry point. The experiment is repeated with the same conditions of charge, initial velocity, mass, etc., but this time a magnetic field is applied perpendicular to the velocity and the particle ends 12cm from the entry point. How far from the entry point, will the particle end, if the magnetic field is halved?

Solution:

In the absence of a magnetic field the acceleration is

$$\frac{d\vec{v}}{dt} = -b\vec{v}$$

Whose solution is

$$\vec{v} = \vec{v}_o \exp(-bt)$$

And integration of this equation will give us the distance traveled from the starting point

$$d = \int_0^{\infty} v_o \exp(-bt)dt = \frac{v_o}{b}$$

Now, when we turn the magnetic field on, the acceleration will be

$$\frac{d\vec{v}}{dt} = -b\vec{v} + \frac{q}{m}\vec{v} \times \vec{B}$$

Since the magnetic field force is orthogonal to the velocity, the speed as a function of time will not change and neither the total distance traveled, but the trajectory will be curved now.

We notice that the velocity will change direction at a constant rate:

$$\frac{d\theta}{dt} = \frac{qB}{m}$$

So, we can write the velocity as a function of time as follows

$$\vec{v} = v_o \left(\cos\frac{qBt}{m}, \sin\frac{qBt}{m} \right) \exp(-bt)$$

And integrating the components $\vec{r} = v_o \left(\int_0^\infty \cos\frac{qBt}{m}\exp(-bt)dt, \int_0^\infty \sin\frac{qBt}{m}\exp(-bt)dt \right)$

The final vector is $\vec{r} = \frac{v_o}{b}\frac{1}{1+\left(\frac{qB}{mb}\right)^2}\left(1, \frac{qB}{mb}\right)$

And its modulus $r = \frac{v_o}{b}\frac{1}{\sqrt{1+\left(\frac{qB}{mb}\right)^2}}$

Replacing $d = \frac{v_o}{b}$ we get $r = \frac{d}{\sqrt{1+\left(\frac{qB}{mb}\right)^2}}$

With the values of the problem: $12 = \frac{20}{\sqrt{1+\left(\frac{qB}{mb}\right)^2}} \to \left(\frac{qB}{mb}\right) = \frac{4}{3}$

The distance with half the field is $r_{B/2} = \frac{d}{\sqrt{1+\left(\frac{qB}{2mb}\right)^2}} = \frac{20}{\sqrt{1+\left(\frac{2}{3}\right)^2}} = \frac{60\sqrt{13}}{13}$