Astronomy

Gravity

Newton's equation for gravitational attraction between two point masses is

$$F = G\frac{m_1 m_2}{r^2}$$

Where m_1 and m_2 are the two masses that attract each other, r is the distance between them and G is a constant equal to

$$G = 6.674\times10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2}$$

A mass M will produce an acceleration at a distance r given by:

$$a = G\frac{M}{r^2}$$

Problem 1.- Find the gravitational acceleration due to the Sun, whose mass is 1.99×10^{30}kg, at the location of the Earth's orbit (a distance r = 1AU=1.4959×10^{11}m).

Solution: The acceleration is given by:

$$a = G\frac{m_{Sun}}{r^2} = 6.67\times10^{-11}\,\frac{1.99\times10^{30}}{(1.4959\times10^{11})^2} = 0.00593\text{m/s}^2$$

Notice that this acceleration is equal to the centripetal acceleration of the Earth, so alternatively we could calculate it as:

$$a = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r = \left(\frac{2\pi}{365.25\times24\times3600\text{s}}\right)^2 1.4959\times10^{11}\text{m} = 0.00593\text{m/s}^2$$

Also notice that this acceleration is 1650 times smaller than g (9.8m/s^2).

Problem 1a.- Find the gravitational acceleration due to the Moon, whose mass is 7.35×10^{22}kg, at the location of the Earth. Take the distance between Earth and Moon to be 3.84×10^{8} m.

Solution: The acceleration is given by:

$$a = G\frac{m_{Moon}}{r^2} = 6.67\times10^{-11}\,\frac{7.35\times10^{22}}{(3.84\times10^{8})^2} = 3.33\times10^{-5}\,\text{m/s}^2$$

We notice that this acceleration is much smaller than the one due to the Sun, but the tidal effects caused by the Moon are larger than the Sun. That is because the Moon is much closer, so the difference in acceleration between the side of the Earth closer to the Moon and the side that is farther is larger than for the case of the Sun.

Problem 2.- A star has been observed orbiting a black hole. It has an orbital period of 15 years, and orbital radius of 0.12 second of arc (as seen from the Earth). Take the distance to the black hole to be 8 kpc. Calculate the mass of the black hole and express your answer as a multiple of the mass of the Sun.
Assume that Newton's law of gravity is applicable for this star's orbit.

Solution: The centripetal acceleration of the star is given by the gravitational attraction of the black hole divided by the star's mass

$$F = G\frac{m_{BH}m_{star}}{r^2} \rightarrow a = G\frac{m_{BH}}{r^2}$$

But this can also be written as a function of the angular velocity or the period of the orbit.

$$a = r\omega^2 = G\frac{m_{BH}}{r^2} \rightarrow m_{BH} = \frac{r^3\omega^2}{G} = m_{BH} = \frac{4r^3\pi^2}{GT^2}$$

The period is given in the problem, and to calculate the radius of the orbit we use the distance to the star and the angle

$$r = 8,000 \times 1.5 \times 10^{11}\,\mathrm{m}\frac{0.12''}{1''} = 1.44 \times 10^{14}\,\mathrm{m}$$

The mass of the black hole is

$$m_{BH} = \frac{4(1.44 \times 10^{14})^3\pi^2}{6.67 \times 10^{-11}(15 \times 3600 \times 24 \times 365)^2} = 7.90 \times 10^{36}\,\mathrm{kg}$$

In multiples of the Sun's mass

$$\frac{m_{BH}}{m_{Sun}} = \frac{7.90 \times 10^{36}\,\mathrm{kg}}{1.99 \times 10^{30}\,\mathrm{kg}} = 3.95 \times 10^6$$