Calculus

Slopes

Problem 1.- The position of a car is given by the values in the following list
Time, t (s) = 1, 2, 3, 4, 5
Distance, s (feet) = 0, 16, 35, 71, 112, 179
Estimate the instantaneous velocity at $t = 2$ by averaging the velocities for the intervals [1, 2] and [2, 3].

Solution: First, let us solve the problem as it is stated and then we discuss what it means. To calculate the velocities in the intervals given we take the displacement in each interval (the difference between final and initial values) and divide by the time interval (also final minus initial values).

$$v_{[1,2]} = \frac{s(2)-s(1)}{t(2)-t(1)} = \frac{35-16}{2-1} = \frac{19}{1} = 19\,feet/s$$

$$v_{[2,3]} = \frac{s(3)-s(2)}{t(3)-t(2)} = \frac{71-35}{3-2} = \frac{36}{1} = 36\,feet/s$$

Now we calculate the average of these two values:

$$v_{average} = \frac{v_{[1,2]}+v_{[2,3]}}{2} = \frac{19+36}{2} = 27.5\,feet/s$$

Notice that the value of position at t=2 is added for the velocity in [1,2], but it is subtracted in [2,3] so it really doesn't affect the average. Also, notice that the final average can be written as follows:

$$v_{average} = \frac{\frac{s(2)-s(1)}{2-1} - \frac{s(3)-s(2)}{3-2}}{2} = \frac{s(3)-s(1)}{2} = 27.5\,feet/s$$

So, the instantaneous value at t=2 is estimated by taking a point after (t=3) minus a point before (t=1). This is the typical way of doing this. Graphically this procedure is equivalent to finding the slope of the "secant" of the curve, which is the red line in the graph.