Calculus

Logarithmic differentiation

Problem 1.- Find the derivatives of the following functions

a) $f(x) = x^2 \ln x$

b) $g(x) = e^x \ln(\tan x) + x^3$

c) $h(x) = \ln(e^{3x} + x^2)$

Solution: We apply the rules of product and compound functions

a) $f'(x) = \left(x^2\right)'\left(\ln x\right) + \left(x^2\right)\left(\ln x\right)$

$$f'(x) = \left(2x\right)\left(\ln x\right) + \left(x^2\right)\left(\frac{1}{x}\right)$$

$$f'(x) = 2x \ln x + x$$

b) $g'(x) = \left(e^x\right)'\left(\ln(\tan x)\right) + \left(e^x\right)\left(\ln(\tan x)\right)' + 3x^2$

$$g'(x) = \left(e^x\right)\left(\ln(\tan x)\right) + \left(e^x\right)\left(\frac{\sec^2 x}{\tan x}\right) + 3x^2$$

$$g'(x) = e^x \ln(\tan x) + 2e^x \csc 2x + 3x^2$$

c) $h'(x) = \dfrac{3e^{3x} + 2x}{e^{3x} + x^2}$

Problem 2.- Find the derivatives of the following functions, use the logarithmic differentiation, if you wish, to simplify your work

a) $y = \sqrt[18]{(x^{10}+1)^3 (x^7 - 3)^8}$

b) $y = \dfrac{e^{3x^2}}{(x^3 - 1)^2 (4x - 7)^3}$

c) $y = x^{\ln\sqrt{x}}$

Solution: We apply logarithmic differentiation

a) First, we take the logarithm on both sides of the equation

$$\ln y = \ln \sqrt[18]{(x^{10}+1)^3 (x^7 - 3)^8}$$

Which can be simplified to give

$$\ln y = \frac{3}{18}\ln(x^{10}+1) + \frac{8}{18}\ln(x^7 - 3)$$

Next, we take derivative on both sides of the equation

$$\frac{y'}{y} = \frac{3}{18}\frac{10x^9}{x^{10}+1} + \frac{8}{18}\frac{7x^6}{x^7-3}$$

And finally, we multiply by y, which we have from the beginning

$$y' = \sqrt[18]{(x^{10}+1)^3(x^7-3)^8}\left(\frac{3}{18}\frac{10x^9}{x^{10}+1} + \frac{8}{18}\frac{7x^6}{x^7-3}\right)$$

b) Taking logarithm

$$\ln y = \ln \frac{e^{3x^2}}{(x^3-1)^2(4x-7)^3}$$

Simplifying

$$\ln y = \ln e^{3x^2} - \ln(x^3-1)^2 - \ln(4x-7)^3$$

$$\ln y = 3x^2 - 2\ln(x^3-1) - 3\ln(4x-7)$$

Taking derivative

$$\frac{y'}{y} = 6x - 2\frac{3x^2}{x^3-1} - 3\frac{4}{4x-7}$$

Multiplying by y

$$y' = \frac{e^{3x^2}}{(x^3-1)^2(4x-7)^3}\left(6x - 2\frac{3x^2}{x^3-1} - 3\frac{4}{4x-7}\right)$$

c) Taking logarithms

$$\ln y = \ln x^{\ln\sqrt{x}}$$

Simplifying

$$\ln y = \ln\sqrt{x}\ln x = \frac{1}{2}\ln x \ln x = \frac{1}{2}\ln^2 x$$

Taking derivative

$$\frac{y'}{y} = \frac{1}{2}2\frac{\ln x}{x} = \frac{\ln x}{x}$$

Multiplying by y

$$y' = \frac{\ln x}{x} x^{\ln \sqrt{x}}$$

Problem 3.- Find y'

$$x^{\sin y} = y^{\cos x}$$

Solution: We apply logarithmic and implicit differentiation

$$\ln x^{\sin y} = \ln y^{\cos x}$$
$$\sin y \ln x = \cos x \ln y$$
$$\cos y y' \ln x + \frac{\sin y}{x} = -\sin x \ln y + \frac{\cos x}{y} y'$$

And solving for y'

$$y' = \frac{y}{x} \frac{\sin y + x \sin x \ln y}{\cos x - y \cos y \ln x}$$