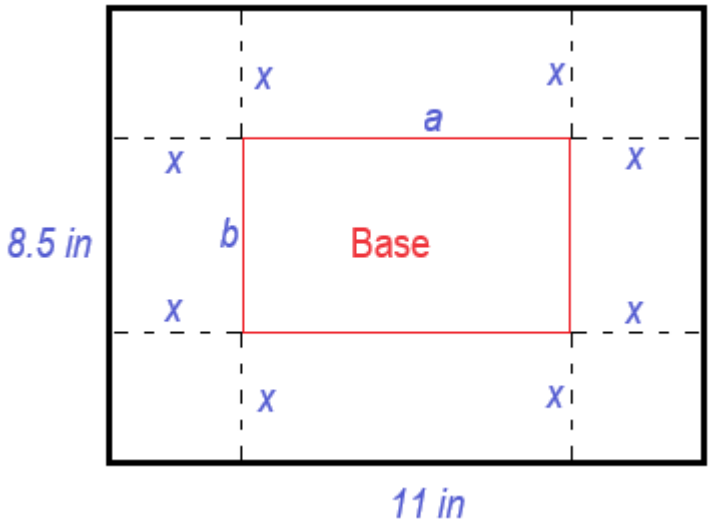
Calculus

Optimization

Problem 1.- A box is constructed from a piece of cardboard 8.5in×11in. Cuts x are made at the corners and the cardboard folded to form the sides. What should be the length x to maximize the volume of the box?



Solution: The volume of the box will be

$$V = abx$$
$$V = (11-2x)(8.5-2x)x$$

Taking the derivative with respect to x:

$$\frac{dV}{dx} = 93.5 - 78x + 12x^2$$
$$12x^2 - 78x + 93.5 = 0$$
$$x = \frac{39 \pm \sqrt{399}}{12} = 1.585\,\text{in}$$
$$V = 66.15$$

Problem 2.- You want to build a corral for cattle by fencing off a rectangular area. One side has a natural barrier, so all you need is the front and the sides. If you have enough materials for 100 meters of fence, what is the maximum area that you can enclose?

Solution:

$$A = xy$$

$$A = x(100 - 2x)$$

$$\frac{dA}{dx} = 100 - 2x - 2x = 0 \rightarrow x = 25$$

$$A_{\max} = 25 \times (100 - 2 \times 25) = 1250$$