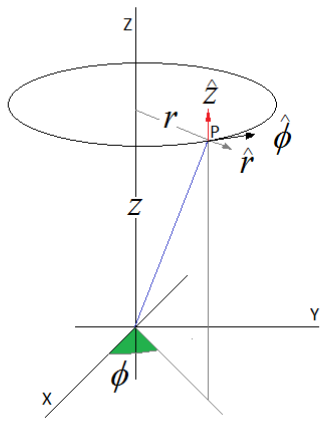
Calculus

Cylindrical Coordinates

In cases studying conducting wires in electromagnetism or waveguides, coaxial cables and many other applications it is convenient to use cylindrical coordinates.

In this type of coordinates, the points in space are specified by a radius r, an angle ϕ and the Cartesian coordinate z, which stays the same.



The relation between the Cartesian and cylindrical coordinates is given by the equations:

$$x = r\cos\phi$$
$$y = r\sin\phi$$
$$r = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$z = z$$

As in Cartesian coordinates, there are unit vectors in the direction of variation of r, ϕ and z that are called $\hat{r}$, $\hat{\phi}$ and $\hat{z}$. They are perpendicular to each other.

For volume integrals in cylindrical coordinates we use the differential:

$$dV = rd\phi drdz$$

Problem 1.- Calculate the total electric charge in a dielectric material that has the shape of a tube with height $h = 10$, with internal radius $r_1 = 2$ and external radius $r_2 = 3$ and whose charge density is given by:

$$\rho = (x^2 + y^2)z$$

Where z is measured from the base of the dielectric.

Solution: We notice that the density can be written in cylindrical coordinates:

$$\rho = r^2 z$$

And the volume integral becomes

$$Q = \int_0^{10}\int_2^3\int_0^{2\pi}(r^2 z)rd\phi drdz$$

We integrate in order

$$Q = \int_0^{10}\int_2^3 2\pi r^3 zdrdz$$

$$Q = \int_0^{10} 32.5\pi zdz$$

$$Q = 1625\pi$$

= **5100**