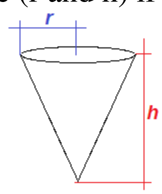
Calculus

Practice Test 1

Problem 1.- A cone for ice cream is made with a thin paste that has a fixed area of 196cm^2. What should be the dimensions of the cone (r and h) if we want to maximize its volume.



The volume of a cone is $V = \frac{1}{3}\pi r^2 h$

The lateral area (without the top circle) is $A = \pi r\sqrt{r^2 + h^2}$

Solution: We can use the area equation to solve for *h*

$$A = \pi r\sqrt{r^2 + h^2} \rightarrow h = \frac{\sqrt{A^2 - \pi^2 r^4}}{\pi r}$$

Then we plug this equation in the volume equation getting

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \frac{\sqrt{A^2 - \pi^2 r^4}}{\pi r} = \frac{1}{3} r\sqrt{A^2 - \pi^2 r^4}$$

Now, we can take the derivative of the volume with respect to *r* and find where it is zero

$$\frac{dV}{dr} = \frac{1}{3}\sqrt{A^2 - \pi^2 r^4} + \frac{1}{3} r \frac{1}{2} \frac{-4\pi^2 r^3}{\sqrt{A^2 - \pi^2 r^4}}$$

$$\frac{1}{3}\sqrt{A^2 - \pi^2 r^4} + \frac{1}{3} r \frac{1}{2} \frac{-4\pi^2 r^3}{\sqrt{A^2 - \pi^2 r^4}} = 0$$

$$A^2 - 3\pi^2 r^4 = 0$$

$$r = \sqrt{\frac{A}{\sqrt{3}\pi}}$$

$$h = \frac{\sqrt{A^2 - \pi^2 \frac{A^2}{3\pi^2}}}{\pi r} = \frac{\sqrt{2}A}{\sqrt{3}\pi r}$$

Problem 2.- Find the minimum of the following function close to x=0.5

$$f(x) = x^5 - x + 1$$

Problem 3.- Find the solution to the following equation close to $x = 1$, stop when the difference between successive approximations if less than 0.000001

$$x - \cot x = 0$$

Problem 4.- Find the limit of the following functions if they exist or explain why they don't

$$a) \lim_{x \to 0} \frac{x^2 \ln x}{\sin x \tan x}$$

$$b) \lim_{x \to 0} \frac{\sqrt{x^2 + x}}{1 - \cos x}$$