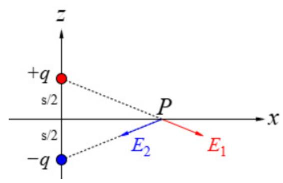
Electromagnetism

Dipole moment

Consider an arrangement of two charges as shown in the figure:



The total charge is zero and this is a common occurrence in nature when a molecule is not symmetric. We are interested in finding the electric field at point P with coordinates (x, 0, 0). This can be done by adding vectors E_1 and E_2. Their magnitudes are the same:

$$\left|\vec{E}_1\right| = \left|\vec{E}_2\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2+(s/2)^2}$$

But when added together we need to take into account their orientation. Notice that the x and y components of the resultant electric field are zero in this case, there is only the z-component:

$$\vec{E}_1 + \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2+s^2/4}\left(0, 0, \frac{-s}{\sqrt{x^2+s^2/4}}\right)$$

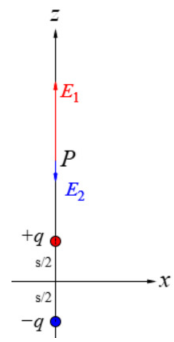
The magnitude of the electric field is:

$$\left|\vec{E}_1 + \vec{E}_2\right| = \frac{1}{4\pi\varepsilon_0}\frac{qs}{x^3\left(1+s^2/4x^2\right)^{3/2}}$$

For values of x much greater than s we can ignore s/x in the denominator and defining the dipole moment as $p = qs$, the electric field can be written as:

$$\left|\vec{E}_1 + \vec{E}_2\right| \approx \frac{1}{4\pi\varepsilon_0}\frac{p}{x^3}$$

Now consider the arrangement shown in the figure, where the coordinates of point *P* are (0, 0, *z*):



This time the electric field will be in the positive *z* direction with a magnitude equal to:

$$\left|\vec{E}_1+\vec{E}_2\right|=\frac{1}{4\pi\varepsilon_0}\frac{q}{(z-s/2)^2}-\frac{1}{4\pi\varepsilon_0}\frac{q}{(z+s/2)^2}$$

This can be rearranged as follows:

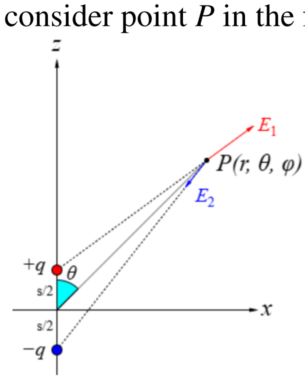
$$\left|\vec{E}_1+\vec{E}_2\right|=\frac{1}{4\pi\varepsilon_0}\frac{q\left(4zs/2\right)}{(z-s/2)^2(z+s/2)^2}=\frac{1}{4\pi\varepsilon_0}\frac{2qs}{z^3(1-s^2/4z^2)^2}$$

In the limit of *z* being much greater than *s*, we can ignore *s*/z in the denominator and substituting $p = qs$ and we get:

$$\left|\vec{E}_1+\vec{E}_2\right|\approx\frac{1}{4\pi\varepsilon_0}\frac{2p}{z^3}$$

By looking at these two examples we notice that the electric field drops inversely proportional to the cube of the distance as opposed as to the square, like for point charges. This makes sense as the net charge is zero, so the fields of the two opposite charges tend to cancel each other.

Now, for a more general case, let us consider point *P* in the figure below:



The electric potential at point *P* is given by:

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{r^2+\left(s/2\right)^2-rs\cos\theta}}-\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{r^2+\left(s/2\right)^2+rs\cos\theta}}$$

If we assume that $s << r$, and define $p = qs$, the expression can be simplified to:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

If we define a dipole moment vector $\vec{p} = \sum \vec{r}_i q_i = sq\hat{z}$, the potential can be written as:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

The electric field can be calculated from the potential by the equation:

$$\vec{E} = -\nabla V$$

In spherical coordinates:

$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

$$E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

$$E_\phi = 0$$