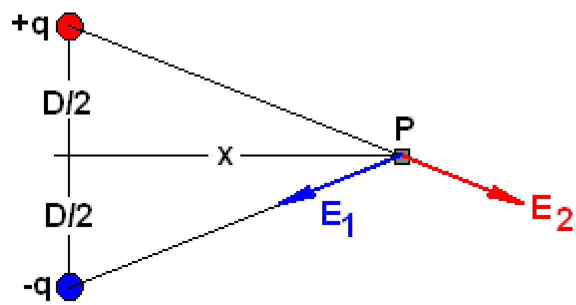
Electromagnetism

Dipole moment

Consider an arrangement of two charges as shown in the figure:



We are interested in finding the electric field at point "P". This can be done by adding the vectors as follows:

$$\left|\vec{E}_1\right| = \left|\vec{E}_2\right| = \frac{kq}{x^2 + (D/2)^2}$$

And: $$\vec{E}_1 + \vec{E}_2 = \frac{kq}{x^2 + (D/2)^2}\left(0, \frac{-2(D/2)}{\sqrt{x^2 + (D/2)^2}}\right)$$

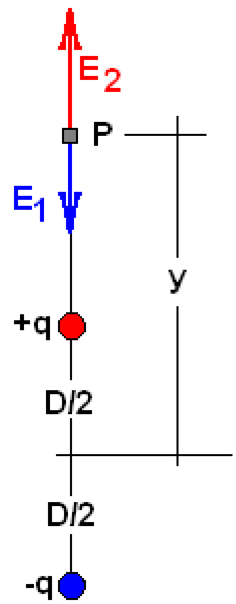
The magnitude of the electric field is:

$$\left|\vec{E}_1 + \vec{E}_2\right| = \frac{kqD}{\left(x^2 + (D/2)^2\right)^{3/2}}$$

For values of x much greater than D we can ignore D in the denominator. Also, the dipole moment is defined as P=qD, so the electric field can be written as:

$$\left|\vec{E}_1 + \vec{E}_2\right| \approx \frac{kP}{x^3}$$

Now consider the arrangement shown in the figure:



This time the electric field will be in the positive y direction with a magnitude equal to:

$$\left|\vec{E}_1 + \vec{E}_2\right| = \frac{kq}{(y-D/2)^2} - \frac{kq}{(y+D/2)^2}$$

This can be rearranged as follows:

$$\left|\vec{E}_1 + \vec{E}_2\right| = \frac{kq(4yD/2)}{(y-D/2)^2(y+D/2)^2} = \frac{2kqyD}{(y-D/2)^2(y+D/2)^2}$$

In the limit of y being much greater than D, we can ignore D in the denominator. Also, we replace P=qD and we get:

$$\left|\vec{E}_1 + \vec{E}_2\right| \approx \frac{2kP}{y^3}$$