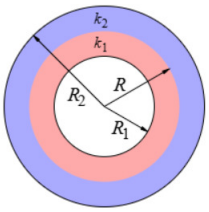
Electromagnetism

Capacitors

Problem 1.- A capacitor is formed by two spherical concentric electrodes of radii R_1 and R_2. The space between them is filled with two materials with dielectric constants k_1 and k_2, in the volumes between R_1 and R and between R and R_2 respectively.
Calculate the capacitance.



Solution: The equation for a spherical capacitor with radii R_1 and R_2 with a dielectric constant k is given by:

$$C = 4\pi\varepsilon_0 k \frac{R_1 R_2}{R_2 - R_1}$$

We can think of the problem as two capacitors in series, the first one formed by the space between R_1 and R, whose capacitance is given by:

$$C_1 = 4\pi k_1 \varepsilon_0 \frac{R_1 R}{R - R_1}$$

And the second formed by the space between R and R_2:

$$C_2 = 4\pi k_2 \varepsilon_0 \frac{R_2 R}{R_2 - R}$$

The total capacitance is:

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{4\pi\varepsilon_0}{\frac{1}{k_1 R_1} - \frac{1}{k_2 R_2} + \frac{1}{R}\left(\frac{1}{k_2} - \frac{1}{k_1}\right)}$$