Electromagnetism

Magnetic dipole

Problem 1.- Consider a thin ring with charge q, mass m and radius R that rotates about its axis with angular velocity ω. Calculate the ratio between its magnetic dipole moment and its angular momentum.
Knowing that the electron has an intrinsic angular momentum h/4π, what would be its magnetic moment if the same ratio could be applied?

Solution: The moment of inertia of a thin ring rotating about its center is $I = mR^2$ and its angular momentum is:

$$L = I\omega = mR^2\omega$$

On the other hand to calculate the magnetic dipole moment we multiply the area enclosed by the ring by the electric current. The electric current can be calculated noticing that the total charge of the ring q will pass through one point in a time equal to 2π/ω, so the dipole is:

$$\mu = \left(\pi R^2\right)\left(\frac{q}{t}\right) = \left(\pi R^2\right)\left(\frac{q}{2\pi/\omega}\right) = \frac{qR^2\omega}{2}$$

The ratio of the magnetic dipole moment to the angular momentum is:

$$\frac{\mu}{L} = \frac{q}{2m}$$

You might find it remarkable that the radius of the ring is not important. You can also demonstrate that the shape of the object doesn't matter, a ring, a sphere, a shell, will give the same result. The only requirement is that the mass density has to be proportional to the charge density.

If we could apply this result to the electron its magnetic moment would be:

$$\mu = L\frac{q}{2m} = \frac{h}{4\pi}\frac{q}{2m} = \frac{6.63\times10^{-34}\,\mathrm{J}s}{4\pi}\frac{1.602\times10^{-19}\,\mathrm{C}}{2\times9.11\times10^{-31}\,\mathrm{kg}} = 4.63\times10^{-24}\,\mathrm{Am}^2$$

This result is almost exactly half the actual value, which is called the Bohr magneton.