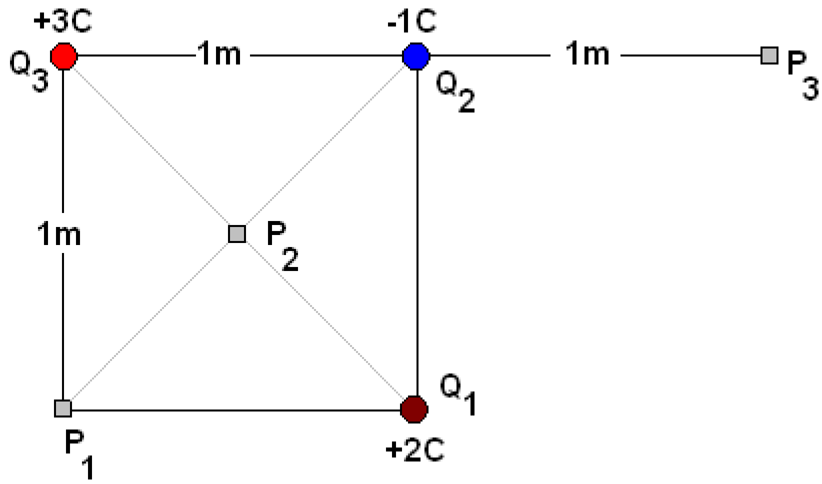
Electromagnetism

Electric potential

Problem 1.-



a) Find the electric potential at points P_1, P_2 and P_3 produced by the three charges Q_1, Q_2 and Q_3.

b) Are there any points where the electric potential is zero? Indicate them in the figure (approximately) and explain your reasoning.

c) Are there any points where the electric field is zero? Indicate them in the figure (approximately) and explain your reasoning.

d) What is the total potential energy stored by the three charges?

Solution:

a) The electric potential is a scalar, so we only need to add the contribution from each charge:

At point P_1

$$V = \frac{kQ_1}{1} + \frac{kQ_2}{\sqrt{2}} + \frac{kQ_3}{1} = k\left(\frac{Q_1}{1} + \frac{Q_2}{\sqrt{2}} + \frac{Q_3}{1}\right) = k\left(\frac{2}{1} + \frac{-1}{\sqrt{2}} + \frac{3}{1}\right) = \mathbf{3.86\times10^{10}\ volts}$$

At point P_2

$$V = \frac{kQ_1}{\sqrt{2}/2} + \frac{kQ_2}{\sqrt{2}/2} + \frac{kQ_3}{\sqrt{2}/2} = \frac{k}{\sqrt{2}/2}\left(Q_1 + Q_2 + Q_3\right) = \frac{k}{\sqrt{2}/2}\left(2 - 1 + 3\right) = \mathbf{5.09\times10^{10}\ volts}$$

At point P_3

$$V = \frac{kQ_1}{\sqrt{2}} + \frac{kQ_2}{1} + \frac{kQ_3}{2} = k\left(\frac{Q_1}{\sqrt{2}} + \frac{Q_2}{1} + \frac{Q_3}{2}\right) = k\left(\frac{2}{\sqrt{2}} + \frac{-1}{1} + \frac{3}{2}\right) = \mathbf{1.72\times10^{10}\ volts}$$

b) For the electric potential to be zero we have to be closer to the negative charge than to the positive ones. This is because the sum of potentials has to be zero, so:

$$V = \frac{kQ_1}{d_1} + \frac{kQ_2}{d_2} + \frac{kQ_3}{d_3} = k\left(\frac{2}{d_1} + \frac{-1}{d_2} + \frac{3}{d_3}\right) = 0 \rightarrow \frac{2}{d_1} + \frac{3}{d_3} = \frac{1}{d_2}$$

The points that satisfy the equation are shown in figure 1.

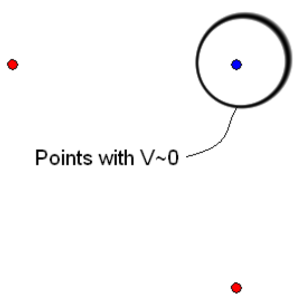


Figure 1

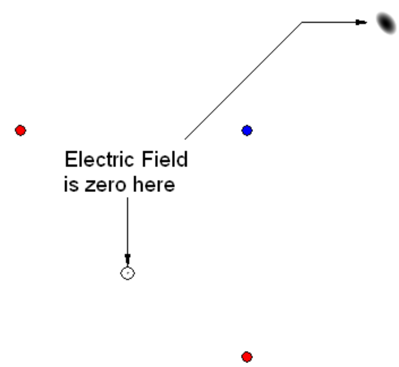


Figure 2

c) For the electric field to be zero the two components of the vectors would need to be zero. Assuming an origin at the position where the negative charge is, the electric field for a point at (x, y) is given by:

$$E = k\left(-1\frac{(x, y)}{\left(x^2 + y^2\right)^{3/2}} + 2\frac{(x, y+1)}{\left(x^2 + (y+1)^2\right)^{3/2}} + 3\frac{(x+1, y)}{\left((x+1)^2 + y^2\right)^{3/2}}\right)$$

There are two conditions for the electric field to be zero:

$$E_x = k\left(\frac{-x}{\left(x^2 + y^2\right)^{3/2}} + \frac{2x}{\left(x^2 + (y+1)^2\right)^{3/2}} + \frac{x+3}{\left((x+1)^2 + y^2\right)^{3/2}}\right) = 0$$

and

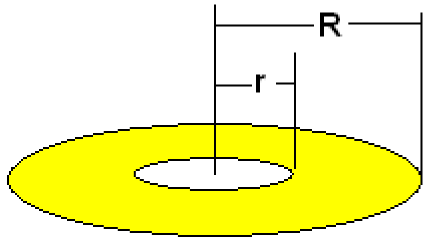
$$E_y = k\left(\frac{-y}{\left(x^2 + y^2\right)^{3/2}} + \frac{2y+2}{\left(x^2 + (y+1)^2\right)^{3/2}} + \frac{3y}{\left((x+1)^2 + y^2\right)^{3/2}}\right) = 0$$

Figure 2 shows where these two points are.

d) The total potential energy stored by the three charges:

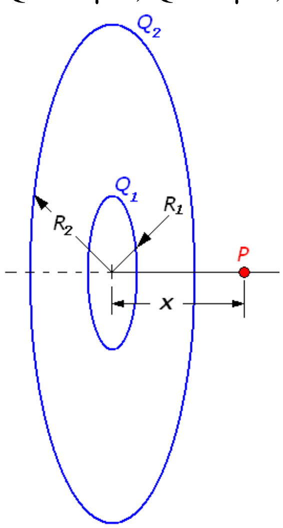
$$Energy = \frac{kQ_1Q_2}{1} + \frac{kQ_1Q_3}{\sqrt{2}} + \frac{kQ_2Q_3}{1} = \frac{-2k}{1} + \frac{6k}{\sqrt{2}} + \frac{-3k}{1} = \mathbf{-6.82\times10^{9}\ J}$$

Problem 2.- Find the electric potential at the center of a flat disk of radius R with a circular hole in the middle of radius r, knowing that the disk has a uniform surface charge density σ



Solution: $V = \int_r^R k \frac{2\pi x \sigma dx}{x} = k2\pi\sigma(R-r)$

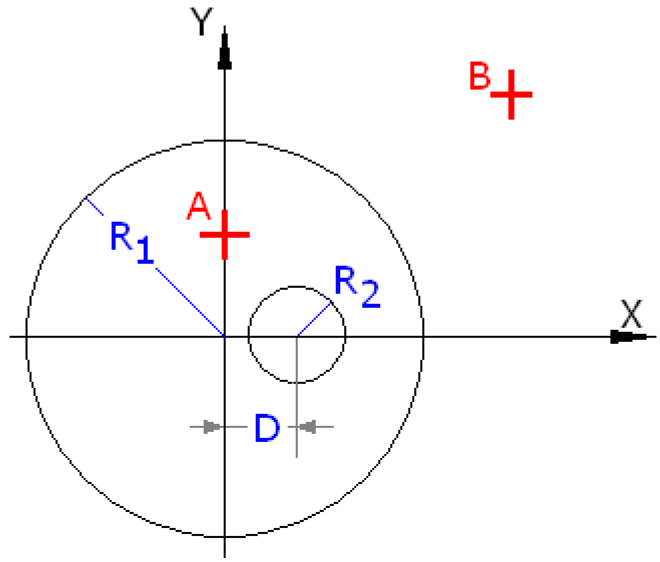
Problem 3.- Find the electric potential at point P, which is on the axis of symmetry of the two uniformly charged rings shown in the figure.
Q_1=2.5μC, Q_2=13μC, R_1=0.15m, R_2=0.48m, X=0.2m



Solution: Since electric potential is a scalar and all the charges in the ring are at the same distance it is the same as if all the charge of the ring where at a point.

$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{Q_1}{\sqrt{R_1^{\,2}+x^2}} + \frac{1}{4\pi\varepsilon_\circ}\frac{Q_2}{\sqrt{R_2^{\,2}+x^2}} = 9\times10^9\left(\frac{2.5\times10^{-6}}{\sqrt{0.15^2+0.2^2}} + \frac{13\times10^{-6}}{\sqrt{0.48^2+0.2^2}}\right) = 3.15\times10^5 V$$

Problem 4.- A sphere of radius R_1=1m is located with its center at the origin of coordinates and has a charge of Q_1=4nC uniformly distributed over its surface. Another sphere of radius R_2=0.3m has its center on the x-axis at a distance of D=0.4m from the origin of coordinates and has a charge of Q_2=-3nC also uniformly distributed over its surface.
Calculate the electric potential at point A=(0,0.5m,0)



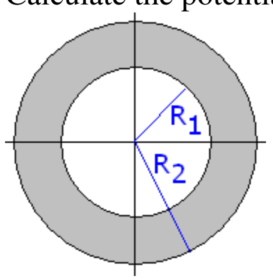
Solution: There are two contributions, the one due to the large sphere is $\frac{kQ_1}{R_1}$ and the one due to the smaller sphere $\frac{kQ_2}{d}$, where d is the distance between point A and the center of the small sphere.

So the potential is: $V = \frac{kQ_1}{R_1} + \frac{kQ_2}{d} = 9\times10^9\left(\frac{4\times10^{-9}}{1} + \frac{-3\times10^{-9}}{\sqrt{0.5^2+0.4^2}}\right)$ **=-6.2 volts**

Notice that the contribution to the potential due to the large sphere is $\frac{kQ_1}{R_1}$, not $\frac{kQ_1}{0.5}$

Problem 5.- A spherical shell has internal radius R_1 and external radius R_2 and contains a uniform distribution of charge with density $\rho = 1.2\mu C/m^3$

Calculate the potential at the center of the shell.



Solution: We realize that the electric field when $r<R_1$ is zero, so the potential at the center is the same as at a distance R_1 so: $V(R_1) = V(0)$

Also, outside the shell the electric potential is the same as if all the charge were located at the center of the sphere (recall Gauss's law) so:

$$V(R_2) = \frac{kQ}{R_2}$$

Where $Q = \frac{4}{3}\pi\left(R_2^{\ 3} - R_1^{\ 3}\right)\rho$

Given this, all we need to do is find the difference in potential between R_2 and R_1, which we can do integrating the electric field.

$$V(R_1) - V(R_2) = -\int_{R_2}^{R_1} E dr$$

The electric field can be obtained by integration:

$$\text{E} = \frac{4}{3}\pi k\rho\left(r - R_1^{\ 3}/r^2\right)$$

Then we calculate the potential:

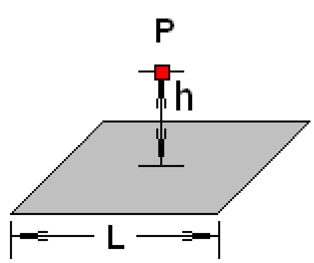
$$V(R_1) = V(R_2) + \int_{R_1}^{R_2} \frac{4}{3}\pi k\rho\left(r - R_1^{\ 3}/r^2\right)dr = V(R_2) + \frac{4}{3}\pi k\rho\left(\frac{r^2}{2} + \frac{R_1^{\ 3}}{r}\right)\Bigg|_{R_1}^{R_2}$$

$$= \frac{k}{R_2}\frac{4}{3}\pi\left(R_2^{\ 3} - R_1^{\ 3}\right)\rho + \frac{4}{3}\pi k\rho\left(\frac{R_2^{\ 2}}{2} + \frac{R_1^{\ 3}}{R_2}\right) - \frac{4}{3}\pi k\rho\left(\frac{R_1^{\ 2}}{2} + \frac{R_1^{\ 3}}{R_1}\right)$$

$V(0) = 2\pi k\rho\left(R_2^{\ 2} - R_1^{\ 2}\right)$=**51,000 volts**

Alternative way: Divide the shell in thin layers of radius *r* and thickness *dr*, so the surface area of a layer is $4\pi r^2$ and the volume is $4\pi r^2 dr$, then the charge of each layer is $4\pi r^2 \rho dr$ and will produce a potential at the center of $\frac{k4\pi r^2 \rho dr}{r} = k4\pi\rho r dr$, finally we integrate from R_1 to R_2 to find the total: $V = \int_{R_1}^{R_2} k4\pi\rho r dr = 2\pi k\rho\left(R_2^{\ 2} - R_1^{\ 2}\right)$

Problem 6.- Find the electric field at point P a distance h=1.2m above the center of a square of side L=2.4m and constant charge density $\sigma = 3.4\mu C/m^2$



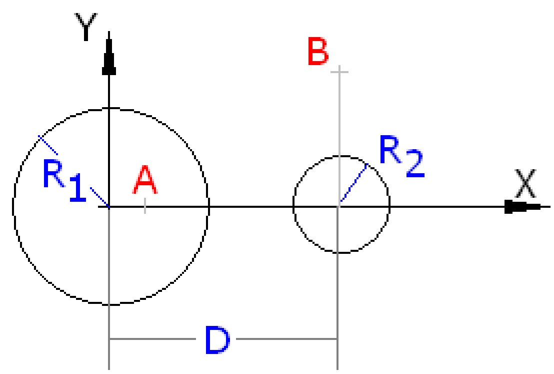
Solution: We learn that the electric field is $E = k\sigma(S.A.)$ where $S.A.$ is the solid angle that the surface presents to point P.

The solid angle in this case is $S.A. = \frac{4\pi}{6}$, which you can calculate by symmetry if you realize the square and point P are one face and the center of a cube.

Then the electric field is: $E = k\sigma\frac{4\pi}{6} =$ **64,100 V/m**

Problem 7.- A sphere of radius R_1=12cm is located with the center at the origin of coordinates and has a charge of 4nC uniformly distributed over its surface. Another sphere of radius R_2=6cm has its center on the x-axis at a distance of D=25cm from the origin of coordinates and has a charge of -3nC also uniformly distributed over its surface.

Calculate the electric potential at point B=(25,15,0)



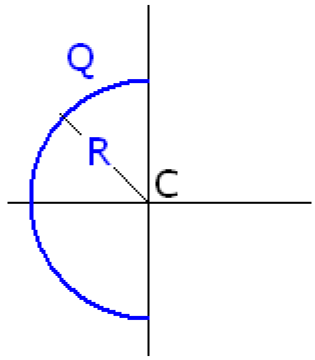
Solution:

The electric potential at point B=(25,15,0): In this case we need to calculate the contribution to the electric potential due to both spheres. When outside a spherical distribution of charges we can calculate the potential contribution of the sphere as if all its charge were located at the center, and so the electric potential at point B is:

$$V = k\frac{q_1}{d_1} + k\frac{q_2}{d_2} = 9\times10^9\frac{4\times10^{-9}}{\sqrt{0.25^2+0.15^2}} + 9\times10^9\frac{-3\times10^{-9}}{0.15} = \textbf{-56.5 V}$$

Notice that for electric potential we do not need to deal with vectors. Potential is just a scalar and the values are either positive or negative depending on the sign of the charges.

Problem 8.- Calculate the electric potential at point "C" due to the uniformly distributed charge Q on the semicircle of radius R.



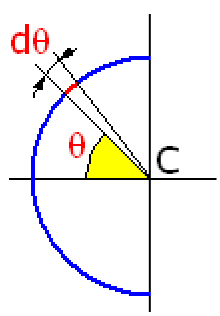
Solution: In problems where there is a continuous distribution of charges the standard procedure to find electric field or potential is:

- Divide the continuous distribution in small pieces (differentials) that can be treated as point charges.
- Calculate the field or potential produced by the differential. Be careful that in the case of electric field you will need to calculate components independently.
- All distances, charges and angles should be written in terms of the variables chosen.
- Integrate over those variables.

The problem above falls precisely in the category that we just mentioned. So let's divide the charge in small pieces. To do this, notice that the charge is uniformly distributed, so the linear density of charge is:

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{\pi R}$$

Then, we divide the arc in differentials as shown in the figure:



The length of the differential of arc is $d\ell = Rd\theta$ and the differential of charge is $dq = \lambda d\ell = \lambda R d\theta = \frac{Q}{\pi R} R d\theta = \frac{Q}{\pi} d\theta$

Now we are ready to find the electric potential. Notice that this differential of charge, taken as if it were a point charge, contributes an electric potential equal to:

$$dV = \frac{kdq}{d} = \frac{k\frac{Q}{\pi}d\theta}{R} = \frac{kQd\theta}{\pi R}$$

And we now integrate to find the potential: $V = \int dV = \int_{-\pi/2}^{\pi/2} \frac{kQd\theta}{\pi R} = \frac{kQ\pi}{\pi R} = \frac{kQ}{R}$

The result is the same as if all the charge had been at a point a distance R away. This is not surprising because all the charge is indeed at the same distance.