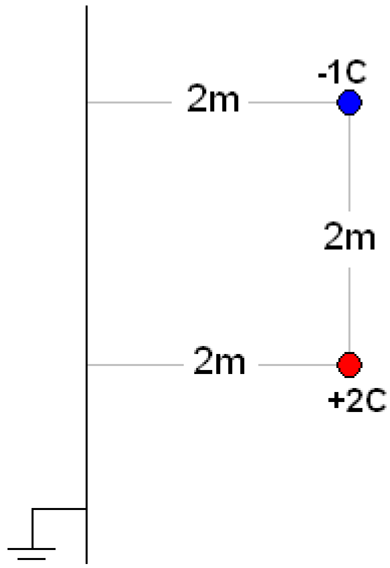
Electromagnetism

Method of images

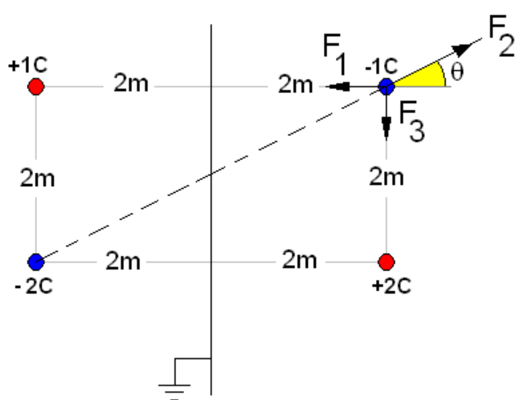
Problem 1.- Consider the two point charges shown in figure 1. They are located in front of a large, flat, conductor surface held at zero potential.

a) Find the net electric force on the negative point charge.

b) Calculate the electric potential energy stored by the system.



Solution:



a) The net electric force on the negative point charge. We replace the infinite plane with two mirror image charges as shown above. Then, to find the force on the negative charge we consider three forces:

$$F_1 = 9\times10^9 \frac{(1)(1)}{4^2} = 5.625\times10^8 N$$

$$F_2 = 9\times10^9 \frac{(1)(2)}{20} = 9\times10^8 N$$

$$F_3 = 9\times10^9 \frac{(1)(2)}{2^2} = 4.5\times10^9 N$$

Since they are vectors, we need to write the vectors as components before adding them. In the case of F_2 notice that the angle is $\theta = \tan^{-1}(2/4) = 26.5^\circ$ so:

$$\vec{F}_1 = (-5.625\times10^8, 0)$$

$$\vec{F}_2 = (9\times10^8\cos\theta, 9\times10^8\sin\theta) = (8.05\times10^8, 4.02\times10^8)$$

$$\vec{F}_3 = (0, -4.5\times10^9)$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (2.42\times10^8, -4.10\times10^9)$$

The magnitude of the force is: $\left|\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right| = \sqrt{(2.42\times10^8)^2 + (4.10\times10^9)^2}$ = **4.1×10^9 N**

The angle of the force is: $Angle = \tan^{-1}\left(\frac{-4.10\times10^9}{2.42\times10^8}\right)$ = **-86º**

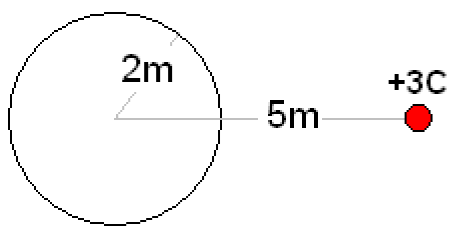
b) Calculate the electric potential energy stored by the system:

$$Energy = \frac{1}{2}k\left(\frac{-1}{4} + \frac{2}{\sqrt{20}} + \frac{-2}{2} + \frac{-2}{2} + \frac{2}{\sqrt{20}} + \frac{-4}{4}\right) = \mathbf{-1.06\times10^{10}J}$$

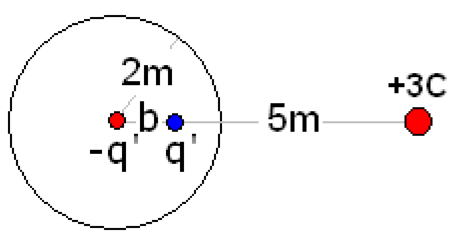
Problem 2.- Consider the point charge and the neutral sphere shown in figure 2.

a) Find the net electric force on the point charge.

b) Calculate the electric potential at the center of the sphere.



Solution: We introduce two image charges as shown below.



a) The net electric force on the point charge.

One image charge is located at the center and the other at a distance "b" from the center. The values are:

$$q' = -\frac{qR}{a} = -\frac{3\times 2}{5} = -1.2C$$
$$b = \frac{R^2}{a} = \frac{2^2}{5} = 0.8m$$

The charges will produce two forces, one attractive and one repulsive. The sum of the vectors is going to be attractive, and the magnitude is:

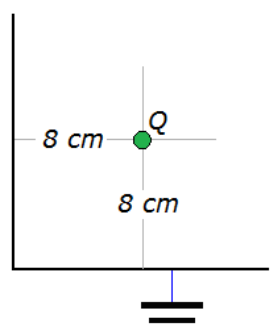
$$F = \frac{9\times 10^9 (3)(1.2)}{(5-0.8)^2} - \frac{9\times 10^9 (3)(1.2)}{5^2} = \mathbf{5.41\times 10^8\ N}$$

b) The potential at the center of the sphere. It is the same as on the surface, which is given by:

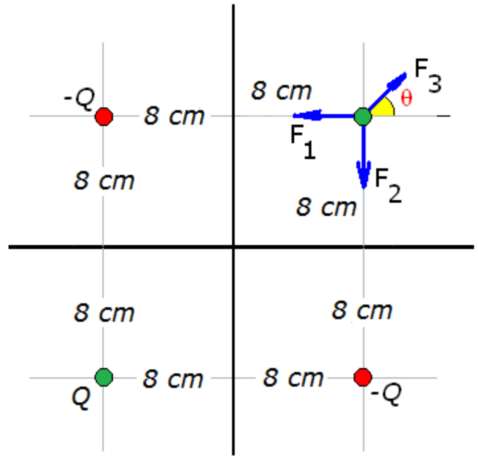
$$V = k\frac{-q'}{R} = \frac{9\times 10^9 (1.2)}{2} = \mathbf{5.4\times 10^9\ volts}$$

Notice that the contributions from q and q' cancel each other at the surface, so we only need to consider -q' which is located at the center.

Problem 3.- A point charge Q=1.2nC is in the proximity of two grounded planes that make an angle of 90 degrees. The particle is 8cm from each plane. Calculate the net force on the particle.



Solution: We can replace the effect of the grounded planes with three image charges as shown:



There will be three forces acting on the particle:

$$F_1 = k\frac{(1.2\times10^{-9})(1.2\times10^{-9})}{0.16^2} = 5.06\text{x}10^{-7}\text{ N} \quad \vec{F}_1 = (-5.06\times10^{-7}, 0)$$

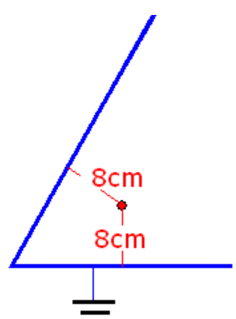
$$F_2 = k\frac{(1.2\times10^{-9})(1.2\times10^{-9})}{0.16^2} = 5.06\text{x}10^{-7}\text{ N} \quad \vec{F}_2 = (0, -5.06\times10^{-7})$$

$$F_3 = k\frac{(1.2\times10^{-9})(1.2\times10^{-9})}{(\sqrt{2}\times0.16)^2} = 2.53\text{x}10^{-7}\text{ N} \quad \vec{F}_3 = (1.79\times10^{-7}, 1.79\times10^{-7})$$

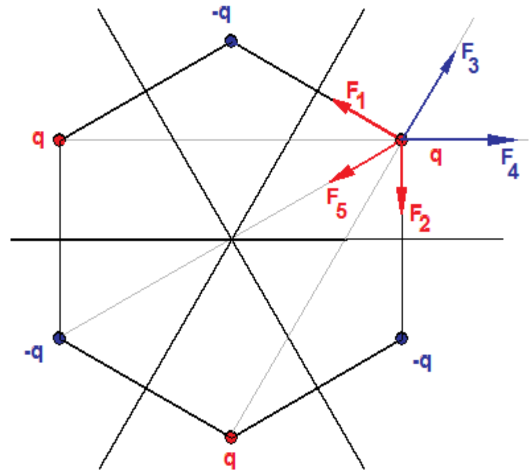
Adding the vectors (after writing them as components)

$$\vec{F} = (-3.27\times10^{-7}, -3.27\times10^{-7})$$

Problem 4.- A 1.2nC point charge is in the proximity of two grounded planes that make an angle of 60 degrees. The particle is 8cm from each plane. Calculate the net force on the particle.



Solution:



With the help of the diagram shown above, we can solve the problem. We replace the effect of the two grounded planes with five image charges at the vertices of a hexagon.

The magnitudes of the forces are:

$$F_1 = F_2 = k\frac{q^2}{0.16^2}$$

$$F_3 = F_4 = k\frac{q^2}{(0.16\sqrt{3})^2} = k\frac{q^2}{3\times 0.16^2}$$

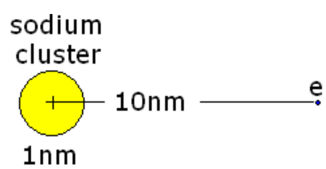
$$F_5 = k\frac{q^2}{(2\times 0.16)^2} = k\frac{q^2}{4\times 0.16^2}$$

By symmetry, we notice that if we add all the vectors the sum will be in the same direction as F_5, so we add the components of all vectors in that direction:

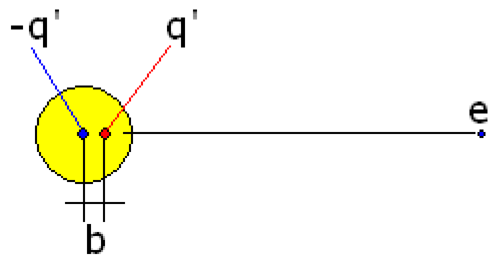
$$F = k\frac{q^2}{4\times 0.16^2} + 2k\frac{q^2}{0.16^2}\cos 60^\circ - 2k\frac{q^2}{3\times 0.16^2}\cos 30^\circ$$

$$F = \frac{kq^2}{0.16^2}\left(\frac{1}{4} + 2\cos 60^\circ - \frac{2}{3}\cos 30^\circ\right) = \mathbf{0.34\ \mu N}$$

Problem 5.- Estimate the force between an electron and a neutral sodium cluster. Assume the sodium cluster is spherical with radius 1nm and behaves as an electric conductor. Take the distance between the center of the cluster and the electron to be 10nm.



Solution: We can solve this problem using the method of image charges. We consider the effect of the electron on the cluster as creating two image charges:



The value of $q' = -\frac{eR}{a} = -\frac{e(1nm)}{10nm} = -e/10$ and the distance $b = \frac{R^2}{a} = \frac{(1nm)^2}{10nm} = 0.1nm$

To find the force we subtract the repulsive force due to –q' from the attractive force of q' as follows:

$$F = k\frac{e(e/10)}{(10nm - 0.1nm)^2} - k\frac{e(e/10)}{(10nm)^2} = \mathbf{4.68{\times}10^{-15}\ N}$$

Note: An alternative way of solving the problem is to calculate the electric field produced by the electron at the location of the sphere: $E = k\frac{e}{a^2}$, then multiply this times the polarizability of the cluster to get the induced dipole: $p = \alpha E = (4\pi\varepsilon_\circ R^3)k\frac{e}{a^2} = \frac{eR^3}{a^2}$ and finally calculate the force between the dipole moment and the charge. See the notes on dipole moments to find the force. Notice the orientation of the dipole, which plays a role in what equation to use.

$$F = \frac{2kpe}{a^3} = \frac{2ke}{a^3}\frac{eR^3}{a^2} = \frac{2kR^3e^2}{a^5} = \mathbf{4.61{\times}10^{-15}\ N}$$