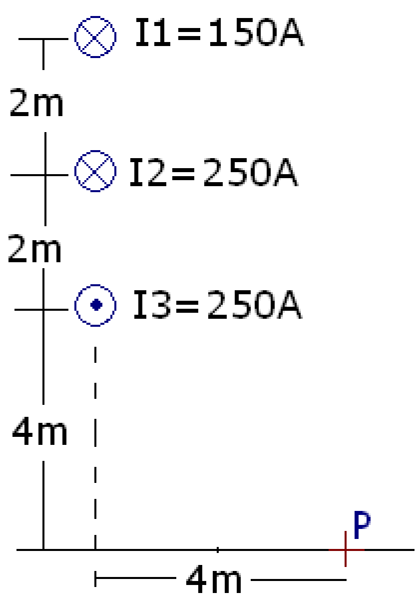
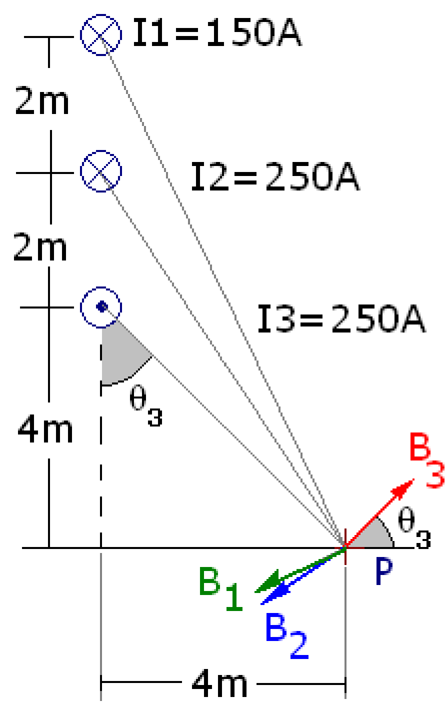
Electromagnetism

Ampere law

Problem 1.- Calculate the magnetic field at point P due to the three long wires whose cross sections are shown in the figure.



Solution: The magnetic field vectors produced by the wires are shown in the figure:



The magnitudes of the vectors are:

$$B_1 = \frac{\mu_\circ I_1}{2\pi R} = \frac{(4\pi\times10^{-7})(150)}{2\pi\sqrt{8^2+4^2}} = \frac{300\times10^{-7}}{\sqrt{80}}T$$

$$B_2 = \frac{\mu_\circ I_2}{2\pi R} = \frac{(4\pi\times10^{-7})(250)}{2\pi\sqrt{6^2+4^2}} = \frac{500\times10^{-7}}{\sqrt{52}}T$$

$$B_3 = \frac{\mu_\circ I_3}{2\pi R} = \frac{(4\pi\times10^{-7})(250)}{2\pi\sqrt{4^2+4^2}} = \frac{500\times10^{-7}}{\sqrt{32}}T$$

We need to write the vectors as components to be able to add them. The geometry of the problem indicates the angles, for example notice that the angle that B_3 makes with the horizontal is the same as the angle between the vertical and the line that connects I_3 with point P (shown in the figure as angle θ_3), likewise with the other angles.

$$\vec{B}_1 = \frac{300\times10^{-7}}{\sqrt{80}}T(-\cos\theta_1, -\sin\theta_1) = \frac{300\times10^{-7}}{\sqrt{80}}T\left(-\frac{8}{\sqrt{80}}, -\frac{4}{\sqrt{80}}\right)$$

$$\vec{B}_2 = \frac{500\times10^{-7}}{\sqrt{52}}T(-\cos\theta_2, -\sin\theta_2) = \frac{500\times10^{-7}}{\sqrt{52}}T\left(-\frac{6}{\sqrt{52}}, -\frac{4}{\sqrt{52}}\right)$$

$$\vec{B}_3 = \frac{500\times10^{-7}}{\sqrt{32}}T(\cos\theta_3, \sin\theta_3) = \frac{500\times10^{-7}}{\sqrt{32}}T\left(\frac{4}{\sqrt{32}}, \frac{4}{\sqrt{32}}\right)$$

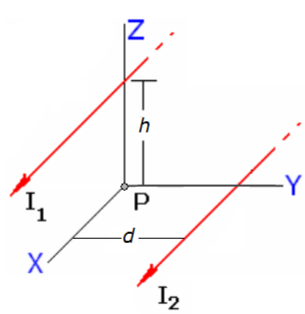
$$\vec{B}_1 = \left(-\frac{24}{80}, -\frac{12}{80}\right)\times10^{-5}T$$

$$\vec{B}_2 = \left(-\frac{30}{52}, -\frac{20}{52}\right)\times10^{-5}T$$

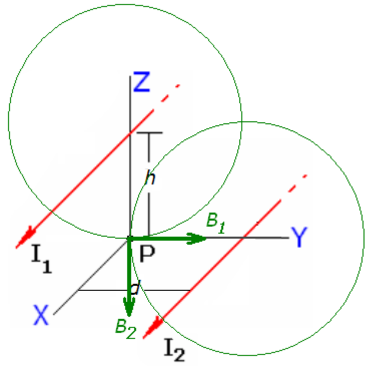
$$\vec{B}_3 = \left(\frac{20}{32}, \frac{20}{32}\right)\times10^{-5}T$$

$$\vec{B}_1 + \vec{B}_2 + \vec{B}_3 = (-0.252, 0.0904)\times10^{-5}T$$

Problem 2.- Two long cables are parallel to the x-axis. One cable is overhead at a height h=9m above the ground and carries a current I_1=810A. The other cable is at ground level, carries a current of I_2=625A and is at a distance d=5m from the X-axis. Calculate the magnetic field at point P.



Solution:

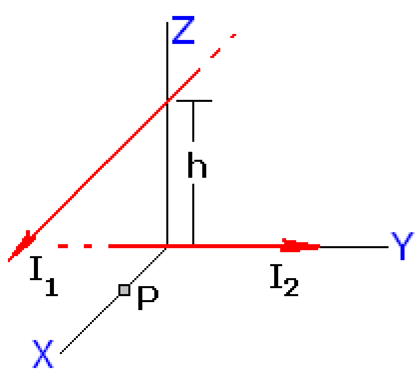


$$B_1 = \frac{\mu_\circ I_1}{2\pi R_1} = \frac{4\pi \times 10^{-7} \times 810}{2\pi \times 9} = 1.8 \times 10^{-5}$$

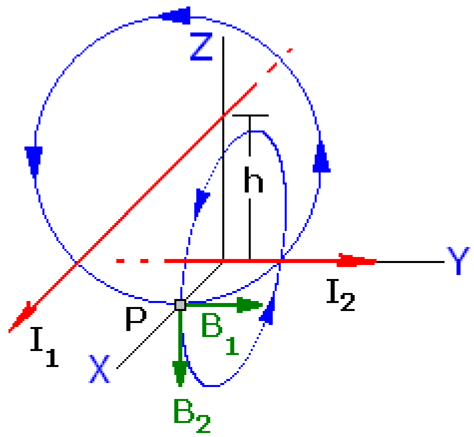
$$B_2 = \frac{\mu_\circ I_2}{2\pi R_2} = \frac{4\pi \times 10^{-7} \times 625}{2\pi \times 5} = 2.5 \times 10^{-5}$$

$$\vec{B} = (0, 18\mu T, -25\mu T)$$

Problem 3.- Two long cables cross each other at 90 degrees as shown in the figure. One cable is parallel to the x-axis but at a height h=10m above the ground and carries a current I_1=150A. The other cable is at ground level, parallel to the y axis and carries a current of I_2=450A. Calculate the magnetic field at point P=(5,0,0)



Solution:



Ampere's law gives us the magnetic field produced by each wire:

$$B_1 = \frac{\mu_\circ I_1}{2\pi r_1} = \frac{4\pi\times10^{-7}\times150}{2\pi r(10)} = 3\mu T$$

$$B_2 = \frac{\mu_\circ I_2}{2\pi r_2} = \frac{4\pi\times10^{-7}\times450}{2\pi r(5)} = 18\mu T$$

The direction of each vector is given by the right hand rule, so the magnetic field is:

$\vec{B} = (0, 3\mu T, -18\mu T)$