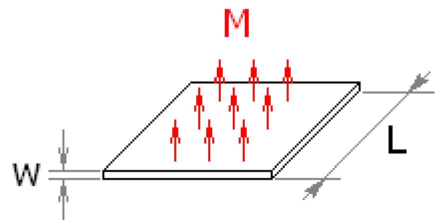
Electromagnetism

Magnets

Problem 1.- A permanent magnet has a square shape of side L and thickness w. It is magnetized uniformly through its thickness with magnetization M. Calculate the intensity of the magnetic field B just above the center of the square magnet. Consider that w<<L.

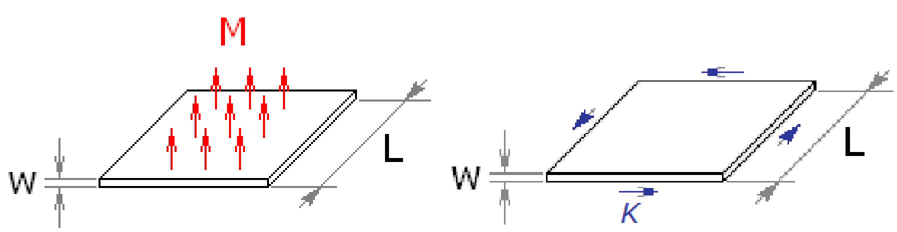


Solution: We can replace the permanent magnet with a distribution of electric current using the equations:

$\vec{J}_b = \nabla \times \vec{M}$, which is a volume current density inside the magnet. It has units of amps per meter squared, so you need to multiply it by the area that the current is going through to get the value of *I*. In the case of this problem, it is zero because the magnetization is uniform.

$\vec{K}_b = \vec{M} \times \hat{n}$, which is also a current density, but a surface current density this time, and it is at the boundary of the magnet. K_b has units of amps per meter and you need to multiply it by the length that the current is going through to get the value of *I*.

We notice that the top surface and bottom surface will give zero K_b because the magnetization and the unit vector point along the same direction (or opposite). On the other hand, the sides of the magnet contribute a value of $K_b = M$ with the direction of the vector as shown in the figure below:



A surface current density $K_b = M$ flowing on the surface of with w means that the current is $I = wM$.

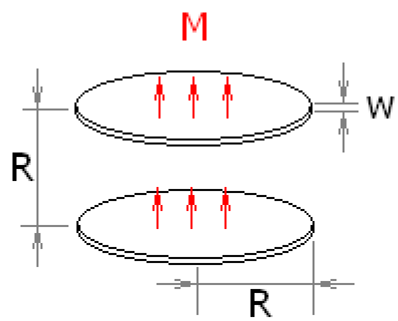
To find the magnetic field at the center of the magnet we can use the equation that we found for a polygon:

$$B = \frac{\mu_\circ I}{2\pi R} N \sin\left(\frac{\pi}{N}\right),$$

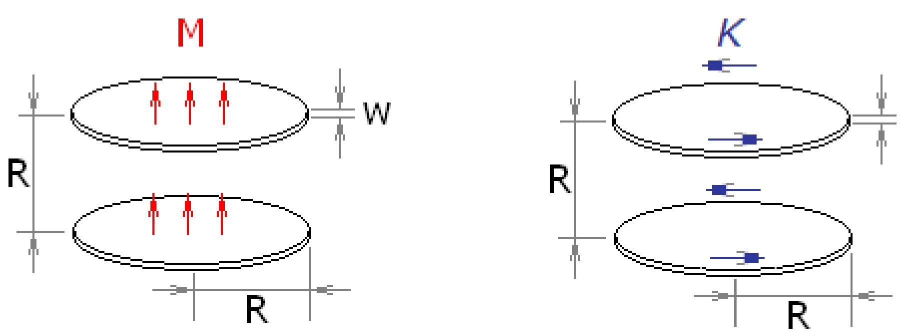
where R is the distance from the center to the side of the polygon and N is the number of sides. In our case N=4, R=L/2 and $I = wM$, so:

$$B = \frac{\mu_\circ wM}{2\pi R} 4\sin\left(\frac{\pi}{4}\right) = \frac{2\sqrt{2}\mu_\circ wM}{\pi L}$$

Problem 2.- Two permanent magnets in the shape of thin disks of radius R and thickness w are separated by a distance equal to the radius R. calculate the magnetic field at the midpoint between the center of the disks. Take w<<R.



Solution: In a similar way as the previous problem we replace the permanent magnets with two circular loops with current $I=Mw$.



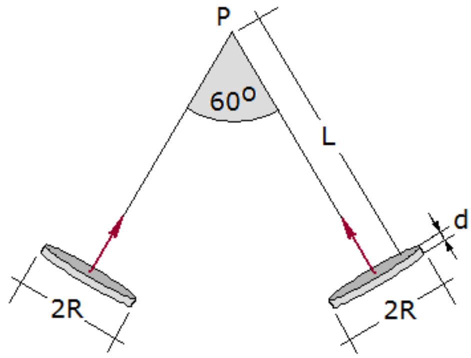
We know how to find the magnetic field above a circular loop with current I. It is the equation:

$$B = \frac{\mu_\circ IR^2}{2(z^2 + R^2)^{3/2}}$$

In this case we substitute $I=Mw$ and $z=R/2$. We also notice that we have two coils, so we need to multiply the magnetic field by two.

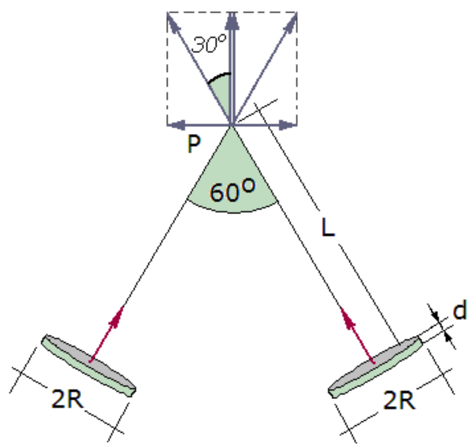
$$B = \frac{\mu_\circ MwR^2}{(R^2/4 + R^2)^{3/2}} = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_\circ Mw}{R}$$

Problem 3.- Two disks of thickness d and radius R, where R>>d, are magnetized through their thickness with uniform magnetization M_o. The disks are arranged symmetrically as shown in figure 1. Calculate the magnetic field produced at point P.



Solution: We substitute the disks with circular loops that carry a current *I=Md*. Each disk produces a field at point P equal to:

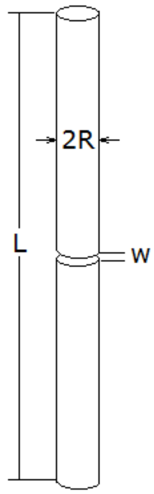
$$B = \frac{\mu_{\circ} I R^2}{2(z^2 + R^2)^{3/2}}$$



In this case we substitute *I=Md* and *z=L*. We also notice that we have two vectors, but only the vertical component needs to be added (the horizontal components cancel each other) so the total field at point P is:

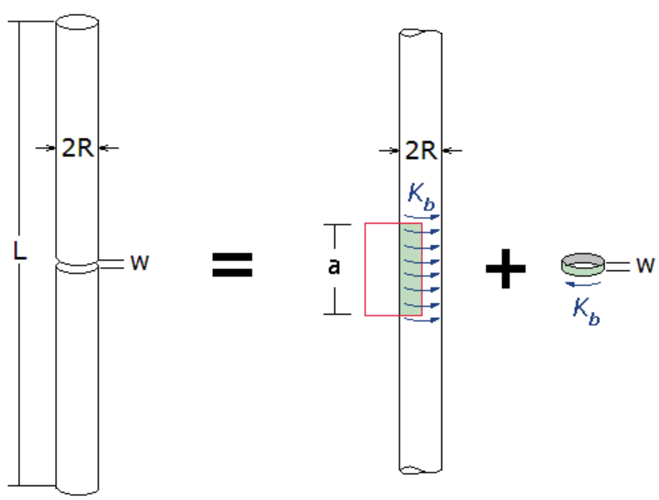
$$B = 2\left(\frac{\mu_{\circ} M d R^2}{2(L^2 + R^2)^{3/2}}\right)\cos 30^{\circ} = \frac{\mu_{\circ} M d R^2 \sqrt{3}}{2(L^2 + R^2)^{3/2}}$$

Problem 4.- A very long rod is magnetized uniformly with magnetization M_o. The rod is cut and a gap w is open as shown in figure 2. Calculate the magnetic field in the middle of the gap if w<<R<<L



Solution: We substitute the long rod with the gap by an infinite solenoid and a circular loop with current in the opposite direction. The surface density of current is $K_b=M$.
To find the effect of the infinite solenoid on the center we use Ampere's law on a simple rectangular circuit as shown in the figure.

$$\oint \vec{B} d\vec{\ell} = \mu_\circ I_{enclosed} \rightarrow Ba = \mu_\circ K_b a = \mu_\circ M a \rightarrow B = \mu_\circ M$$

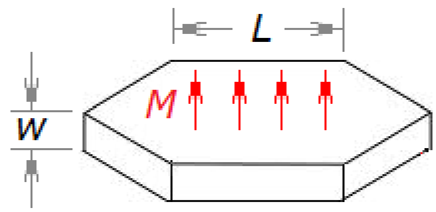


The circular loop that replaces the gap has current in the opposite direction, with a value of $I=MW$. And the magnetic field produced is:

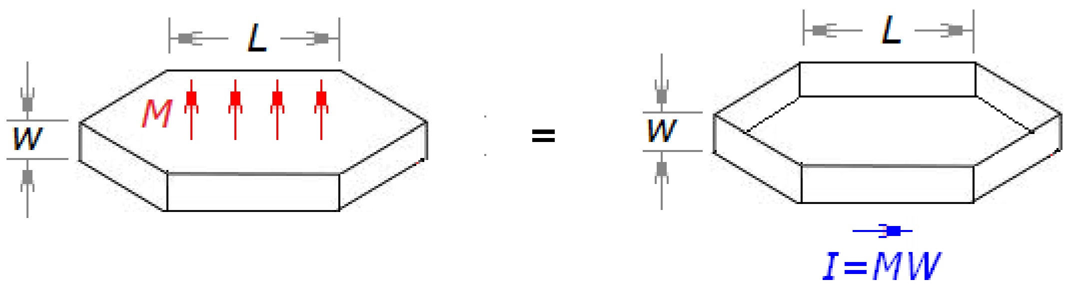
$$B = \frac{\mu_\circ I}{2R} = \frac{\mu_\circ MW}{2R}$$

The total magnetic field is: $B = \mu_\circ M - \dfrac{\mu_\circ MW}{2R} = \mu_\circ M\left(1 - \dfrac{W}{2R}\right)$

Problem 5.- A permanent magnet has an hexagonal shape of side *L* and thickness *W*. It is magnetized uniformly through its thickness with magnetization *M*. Calculate the intensity of the magnetic field B just above its center. Consider that *W*<<*L*.



Solution: We replace the effect of the hexagonal magnet with a hexagonal circuit:



The distance between the center of the hexagon and one side is $R = \dfrac{\sqrt{3}}{2}L$

The magnetic field is: $B = \dfrac{\mu_\circ I}{2\pi R} N \sin\left(\dfrac{\pi}{N}\right) = \dfrac{\mu_\circ MW}{2\pi \dfrac{\sqrt{3}}{2} L} 6 \sin\left(\dfrac{\pi}{6}\right) = \dfrac{\mu_\circ \sqrt{3} MW}{\pi L}$