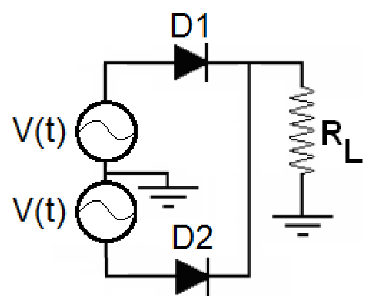
Electronics

Full wave rectifier

One configuration of the full wave rectifier is the one represented by the following schematic:



Let the source voltage be the function:

$$V(t) = V_o \sin \omega t$$

When V(t) is positive, as a first approximation, diode D1 will behave as a short circuit and V(t) will be across the load. When the source voltage is negative, diode D2 will in turn behave as a short and –V(t) will be across the load.
Therefore, the load voltage will be the absolute value of the source:

$$V_L(t) = V_o |\sin \omega t|$$

We can use a Fourier series to represent this voltage obtaining:

$$V_L(t) = \frac{2V_o}{\pi} - V_o \sum_{n=2,even}^{\infty} \frac{4}{\pi(n^2-1)} \cos n\omega t$$

Showing the first few terms explicitly:

$$V_L(t) = V_o \left[\frac{2}{\pi} - \frac{4}{3\pi}\cos 2\omega t - \frac{4}{15\pi}\cos 4\omega t - \frac{4}{35\pi}\cos 6\omega t - \frac{4}{63\pi}\cos 8\omega t + ... \right]$$

From the first term, we notice that the DC component is:

$$V_{DC} = \frac{2V_o}{\pi}$$

Integrating the original function, we get the RMS value of the load voltage:

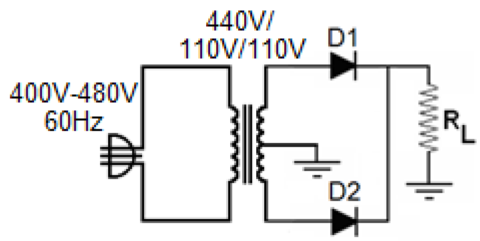
$$V_{RMS} = \frac{V_o}{\sqrt{2}}$$

In the equations above, we could also include the voltage drop in the diode (0.7V for silicon diodes) as a second approximation to be more accurate. If a bridge is used, there will be a drop in two diodes (a total of 1.4V for silicon diodes).

Problem 1.- A factory is at the end of distribution line and there are fluctuations in the voltage between 400V and 480V.
You want to feed DC to a resistive load R_L=12.1ohm using a transformer with a tap in the middle of the secondary, rated 440V/110V/110V and a full wave rectifier as shown in the diagram below.

Specify for the diodes:

a) Maximum average current.
b) Peak current.
c) Maximum reverse voltage.

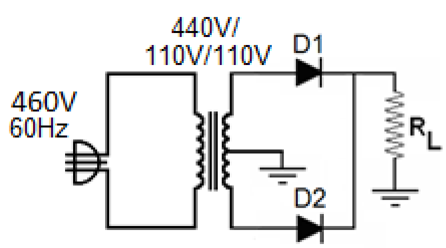


Solution: The worst case scenario happens when the source voltage is 480V, in this case the transformer output will be $\frac{110}{440}\times 480 = 120V$, with this value:

a) Average current in the diodes: $I_{ave} = \frac{1}{2} I_{DC-load} = \frac{1}{2} \frac{\frac{2}{\pi}\times 120\sqrt{2}}{12.1} = 4.46A$
b) Peak current: $I_{max} = \frac{120\sqrt{2}}{12.1} = 14.0A$
c) Peak inverse voltage: $PIV = 2\times 120\sqrt{2} = 339V$

Problem 2.- A full wave rectifier with a resistive load R_L=6.05 ohm is fed from the secondary of a transformer as shown below.

a) Calculate the power dissipated by each diode.
b) Sketch a graph of voltage in diode D1 vs. time.



Problem 3.- Calculate the amplitude of the second harmonic at the output of a full wave rectifier. Give the result compared to the amplitude of the original sine wave.

Solution: Given the full wave rectified signal

$$f = \begin{cases} \sin 2\pi \frac{t}{T} \; 0 < t < T/2 \\ -\sin 2\pi \frac{t}{T} \; T/2 < t < T \end{cases}$$

The amplitude required can be calculated as follows:

$$r = \frac{\int_0^{T/2} \sin 2\pi \frac{t}{T} \cos 2\pi \frac{2t}{T} dt + \int_{T/2}^{T} -\sin 2\pi \frac{t}{T} \cos 2\pi \frac{2t}{T} dt}{\int_0^T \cos^2 2\pi \frac{2t}{T} dt} = \frac{4}{T} \int_0^{T/2} \sin 2\pi \frac{t}{T} \cos 2\pi \frac{2t}{T} dt$$

$$r = \frac{2}{2\pi} \int_0^{\pi} \sin 3x - \sin x dx = \frac{1}{\pi} \left(-\frac{\cos 3x}{3} + \cos x \right)_0^{\pi} = -\frac{4}{3\pi}$$

This is 42% of the original amplitude.

Problem 4.- A full wave rectifier with input 10V at 60Hz is used as a crude frequency doubler. If the output first is connected through a large coupling capacitor and then a low pall filter with one pole at 60Hz, estimate the amplitude of the signal at 120Hz.

Solution: The Fourier series of the full wave rectifier is:

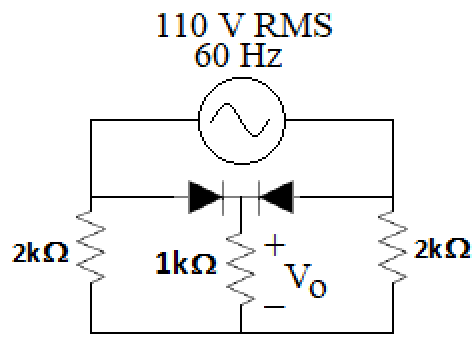
$$V_L(t) = V_o \left[\frac{2}{\pi} - \frac{4}{3\pi} \cos 2\omega t - \frac{4}{15\pi} \cos 4\omega t - \frac{4}{35\pi} \cos 6\omega t - \frac{4}{63\pi} \cos 8\omega t + \ldots \right]$$

The coupling capacitor will eliminate the DC and the filter will reduce the second harmonic to:

$$V_{120Hz} = V_o \frac{4}{3\pi} \frac{1}{\sqrt{1 + \left(\frac{120}{60} \right)^2}} = 0.19 V_o$$

The value of V_o is 10V times $\sqrt{2}$ minus the drop in the diodes. If we use a full wave rectifier with a split secondary transformer we subtract 0.7V and if it is a bridge, we subtract 1.4V.

Problem 5.- Analyze the behavior of the following rectifier with silicon diodes. You can approximate the diodes as ideal.

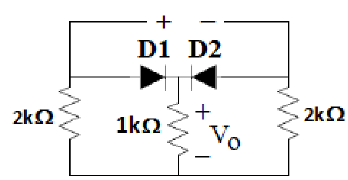


a) Sketch the output voltage V_o as a function of time.
b) Calculate the DC component of V_o.

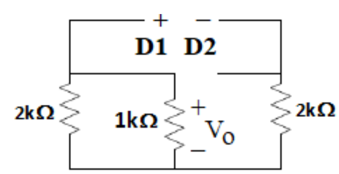
To specify the diodes for this application respond:

c) What is the average current in the diodes?
d) What is the maximum current in the diodes?

Solution: Given that the maximum voltage in the source is $110\sqrt{2} = 156V$ it is indeed possible to use the approximation of ideal diodes without much error.
First we analyze the case when the polarity of the source is as shown below:



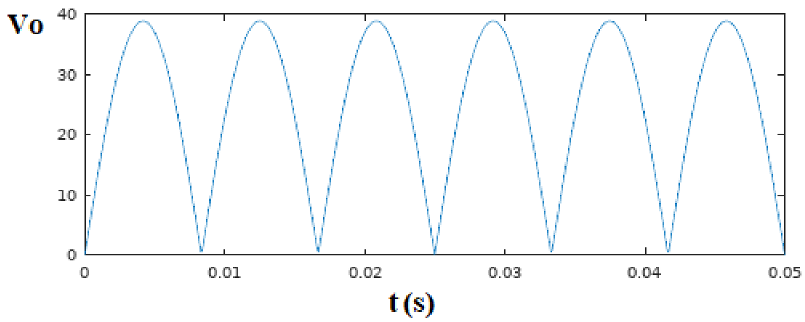
In this case, diode D1 is in conduction and D2 is open, equivalent to this circuit:



The voltage in the load can be calculated by a Thevenin equivalent. The load is in parallel with a 2kohm resistor and this combination is in series with another 2kohm resistor. The voltage in the load is a fraction of the voltage in the source:

$$V_O = V_S \frac{1k\Omega // 2k\Omega}{1k\Omega // 2k\Omega + 2k\Omega} = 0.25V_S$$

When the polarity is inverted, the solution is similar, giving the voltage in the load as sketched below.



The peak voltage in the load is

$$V_{O-peak} = 0.25 \times 110\sqrt{2} = 38.9V$$

And the DC component is

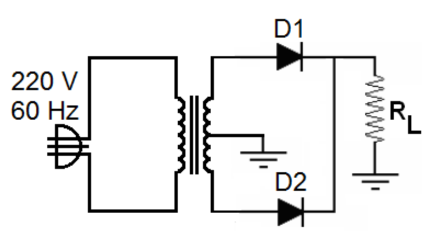
$$V_{O-DC} = \frac{2 \times 38.9V}{\pi} = 24.8V$$

In the diodes the current flows only during one half of the cycle, so the DC current is just one half of the DC current in the load

$$I_{diode-DC} = \frac{38.9V}{\pi \times 1k\Omega} = 12.4mA$$

The peak current in the diodes is the same as in the load: $I_{diode-peak} = \frac{0.25 \times 110\sqrt{2}V}{1k\Omega} = 38.9mA$

Problem 6.- A full wave rectifier as shown in the figure has to be designed to supply a resistive load R_L=100ohm with a DC voltage of 32V. Calculate the load RMS voltage and power, the transformer secondary voltage, maximum peak and average current of the diodes and their maximum reverse voltage.



Solution: Knowing the required DC voltage, we can calculate the RMS voltage using the equations deduced before for the full wave rectifier:

$$V_{DC} = \frac{2V_{\max}}{\pi}$$

$$V_{RMS} = \frac{V_{\max}}{\sqrt{2}}$$

$$\rightarrow V_{RMS} = \frac{\pi V_{DC}}{2\sqrt{2}} = \frac{\pi(32V)}{2\sqrt{2}} = 35.5V$$

Given this RMS voltage, we will need a transformer with a split secondary 35.5V/35.5V

The peak current in the diodes will be

$$I_{peak} = \frac{V_{\max}}{R_L} = \frac{\pi V_{DC}}{2R_L} = \frac{\pi(32V)}{2(100\Omega)} = 503mA$$

The average current in the diodes will be half the DC current of the source

$$I_{ave} = \frac{I_{DC}}{2} = \frac{V_{DC}}{2R_L} = \frac{32V}{2(100\Omega)} = 160mA$$

For the reverse voltage, notice that in this configuration when not conducting, the voltage across the diodes is twice the load voltage, so:

$$V_{reverse} = 2V_{\max} = \pi V_{DC} = \pi(32V) = 101V$$