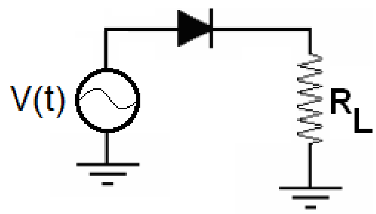
Electronics

Half wave rectifier

The basic circuit is represented by the following schematic:



Let the source voltage be the function:

$$V(t) = V_o \cos \omega t$$

When the source voltage is positive, as a first approximation, the diode will behave as a short circuit and the voltage will be across the load. When the source voltage is negative, no current will flow and the voltage across the load will be zero.
Therefore, the load voltage will be:

$$V_L(t) = \begin{cases} V_o \cos \omega t & when\ 2n\pi - \pi/2 < \omega t < 2n\pi + \pi/2,\ n \text{ integer} \\ 0 & when\ 2n\pi + \pi/2 < \omega t < 2n\pi + 3\pi/2, n \text{ integer} \end{cases}$$

We can use a Fourier series to represent this voltage, obtaining:

$$V_L(t) = \frac{V_o}{\pi} + \frac{V_o}{2}\cos \omega t + V_o \sum_{n=2,even}^{\infty} \frac{2(-1)^{n/2+1}}{\pi(n^2-1)} \cos n\omega t$$

Showing the first few terms explicitly:

$$V_L(t) = V_o\left[\frac{1}{\pi} + \frac{1}{2}\cos \omega t + \frac{2}{3\pi}\cos 2\omega t - \frac{2}{15\pi}\cos 4\omega t + \frac{2}{35\pi}\cos 6\omega t - \frac{2}{63\pi}\cos 8\omega t + \ldots\right]$$

From the first term, we notice that the DC component is:

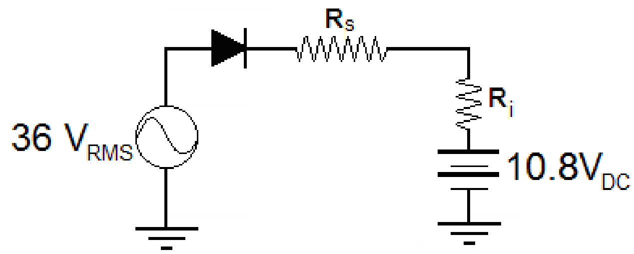
$$V_{DC} = \frac{V_o}{\pi}$$

Integrating the original function, we get the RMS value of the load voltage:

$$V_{RMS} = \frac{V_o}{2}$$

In the equations above, we could also include the voltage drop in the diode (0.7V for silicon diodes) as a second approximation to be more accurate.
Notice that the simple half wave rectifier has a large fraction (50%) of the original fundamental frequency besides the desired DC component.

Problem 1.- To charge a 12V battery you use a half wave rectifier that takes its source voltage at 36V RMS. When fully discharged, this battery is modeled as 10.8V DC source in series with an internal resistance R_i=0.07ohm. Specify a resistor R_S to connect in series to keep the average charging current at 5.0A.



Solution: The circuit is a half wave rectifier, but the battery is not a resistive load, so we will need to consider this later.

The peak value of the voltage source is $V_{\max} = 36\sqrt{2} = 50.9V$

For the diode to start conducting, the voltage in the source will need to be at least 10.8V+0.7V=11.5V and this will cause two effects:

i) The diode will conduct current for less than half a cycle (<180 degrees).
ii) When calculating the current we need to subtract 11.5V from the source voltage.

First, we can approximate the solution ignoring the first effect and just use the equations we deduced earlier, but with 11.5V instead of 0.7V for the voltage drop:

The average voltage is $V_{DC} = \frac{V_{\max}}{\pi} = \frac{50.9V - 11.5V}{3.14} = 12.5V$

To get 5.0A of DC current the resistance is $R_S = \frac{12.5V}{5.0A} - 0.07 = 2.43\Omega$

In addition, the RMS voltage will be $V_{RMS} = \frac{V_{\max}}{2} = \frac{50.9V - 11.5V}{2} = 19.7V$

The RMS current will be $I_{RMS} = \frac{19.7V}{0.07\Omega + 2.43\Omega} = 7.88A$

The power dissipated in the series resistance will be $P_{R_S} = 2.43\Omega(7.88A)^2 = 151W$

In conclusion, we would need to get a series resistor of **2.43Ω** and **151W**

A more accurate calculation should take into account the first effect.

Problem 2.- In a half wave rectifier with input peak voltage of 14V and 60Hz frequency, you are asked to find the amplitude of the fourth harmonic after passing the signal for a low-pass filter with one pole at 120Hz.

Solution: The Fourier series for this wave is

$$V_L(t) = V_o\left[\frac{1}{\pi} + \frac{1}{2}\cos\omega t + \frac{2}{3\pi}\cos 2\omega t - \frac{2}{15\pi}\cos 4\omega t + \frac{2}{35\pi}\cos 6\omega t - \frac{2}{63\pi}\cos 8\omega t + \ldots\right]$$

The fourth harmonic has an amplitude of

$$V_4 = \frac{2}{15\pi}V_o$$

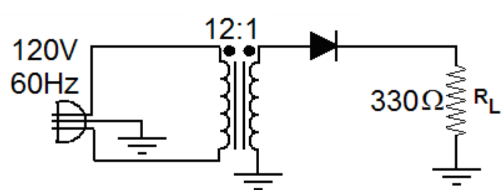
Its frequency is 240Hz and the filter has the cutoff frequency at 120Hz, so the amplitude becomes:

$$V_4 = \frac{2}{15\pi}V_o \frac{1}{\sqrt{1+\left(\frac{240}{120}\right)^2}} = 266\text{mV}$$

Approximations are also acceptable. Notice that 240Hz is one octave higher than the cut-off frequency, so we expect a drop of 6dB (a factor of 2) which gives the approximated 297mV instead of 266mV.

Problem 3.- Using the second approximation for the silicon diode in the following circuit calculate:

a) The DC output voltage (V_{DC})
b) The RMS output voltage (V_{RMS})
c) The power dissipated in the load resistor (P_{RL})



Solution:

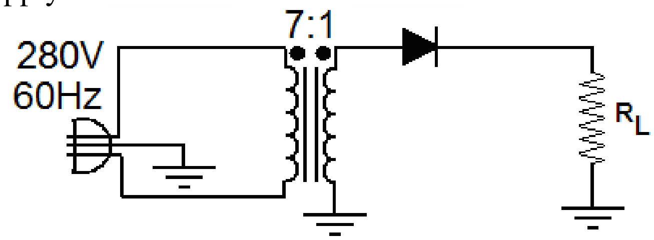
a) The peak value after the transformer is: $V_{peak} = \frac{120}{12}\sqrt{2} = 14.1$ volts and so the DC value is:

$$V_{DC} = \frac{14.1 - 0.7}{\pi} = \mathbf{4.3\ V}$$

b) The RMS value: $V_{RMS} = \frac{14.1 - 0.7}{2} =$ **6.7 V**

c) The power: $P = \frac{V_{RMS}^2}{R} = \frac{(6.7V)^2}{330\Omega} =$ **136 mW**

Problem 4.- In a half wave rectifier as shown below the diode has maximum average current of 1.0A and maximum peak current of 5.0A. With these specifications, what is the minimum load resistance that it can supply?



Solution: We recognize the circuit as a half wave rectifier.

The secondary voltage of the transformer is $V_s = \frac{280V}{7} = 40V$

The maximum voltage of the secondary will be:

$$V_{\max} = 40V\sqrt{2} = 56.6V$$

Since the peak current must be 5.0A maximum, the resistance needs to be at least:

$$R_L \geq \frac{56.6V}{5.0A} = 11.3\Omega$$ … Restriction due to maximum peak current

The DC voltage:

$$V_{DC} = \frac{V_{\max}}{\pi} = \frac{56.6V}{3.14} = 18.0V$$

Since the average current must be 1.0A maximum, the resistance needs to be at least:

$$R_L \geq \frac{18V}{1.0A} = 18\Omega$$ … Restriction due to maximum average current

We conclude that the minimum load resistance must be **18 ohm**.