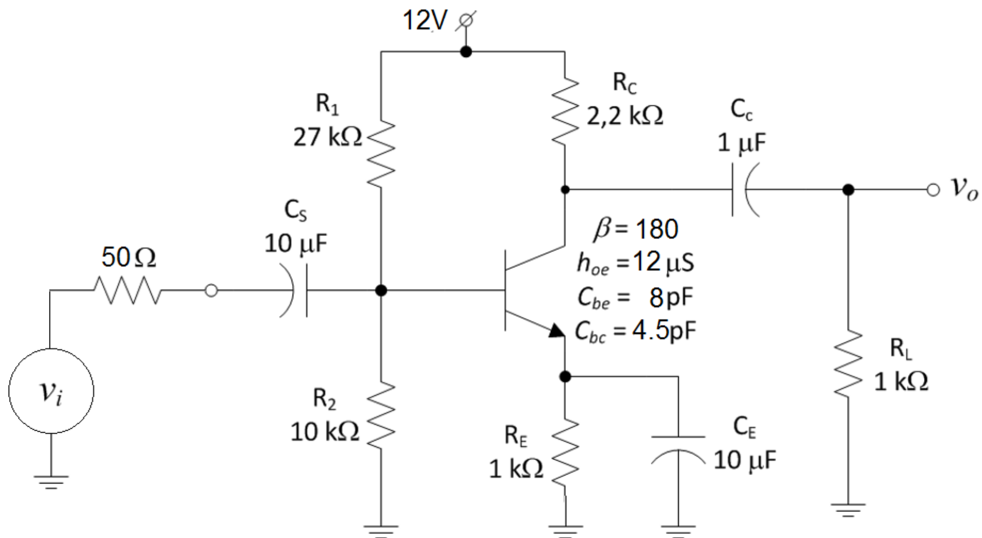
Electronics

Frequency response of BJT amplifiers

Problem 1.- Find the Bode diagram of the frequency response in the following common emitter amplifier with RC coupling.



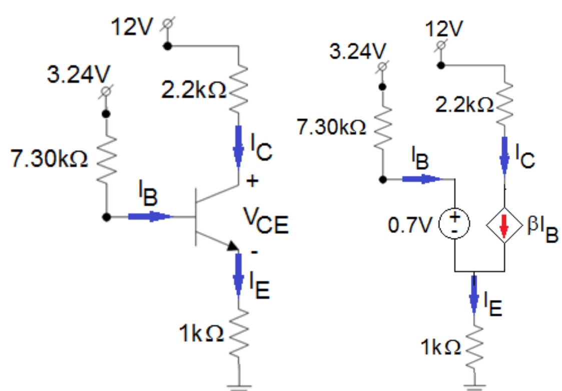
Solution: ***First we calculate the operating point Q and the medium band gain***

The Thevenin equivalent of the biasing circuit gives us

$$V_{Th} = 12\frac{10k}{27k + 10k} = 3.24V$$

$$R_{Th} = \frac{10k \times 27k}{27k + 10k} = 7.30k\Omega$$

With that change the DC circuit will be as shown below on the left.



The diagram above on the right is the DC model of the transistor neglecting h_{oe}.
The Q point is calculated below

$$I_B = \frac{3.24V - 0.7V}{7.3k\Omega + 180 \times 1k\Omega} = 13.5\mu A$$
$$I_C = 180 \times 13.6\mu A = 2.43mA$$
$$I_E = 181 \times 13.6\mu A = 2.44mA$$
$$V_{CE} = 12V - 2.43mA \times 2.2k\Omega - 2.44mA \times 1k\Omega = 4.21V$$

The base-emitter dynamic resistance from the point of view of the base is

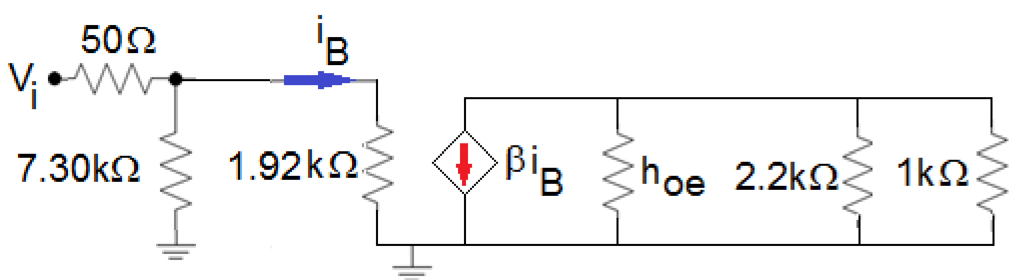
$$r_b = \frac{26mV}{13.5\mu A} = 1.92k\Omega$$

And from the point of view of the emitter

$$r_e = \frac{26mV}{181 \times 13.6\mu A} = 10.6\Omega$$

The value of h_{oe} is 12µS or 1/83.3kohm and was neglected in this calculation.

The AC analysis is done with the following circuit model:



The base current in AC is

$$i_B = v_i \frac{7.3k\Omega}{7.3k\Omega + 50\Omega} \frac{1}{1.92k\Omega + \frac{50\Omega \times 7.3k\Omega}{7.3k\Omega + 50\Omega}} = 5.03 \times 10^{-4} v_i \ldots \text{ equation (*)}$$

And the gain in AC is

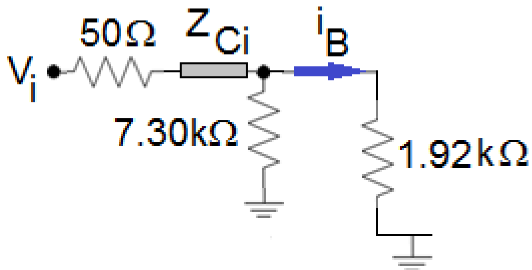
$$\frac{v_o}{v_i} = -180 \times (5.03 \times 10^{-4}) \left(2.2k\Omega // 1k\Omega // \frac{1}{12\mu S} \right) = -61.7$$

Which converted to decibels is

$$20 \log \left| \frac{v_o}{v_i} \right| = 35.8dB$$

Calculation of low cutoff frequencies:

The coupling input capacitor is connected in series with the input impedance of the amplifier. The circuit below shows its place in the circuit.



It has the effect of changing the base current in AC from the value in equation (*) to the following equation

$$i_B' = v_i \frac{1}{50\Omega + \dfrac{1.92k\Omega \times 7.3k\Omega}{7.3k\Omega + 1.92k\Omega} + Z_{Ci}} \frac{7.3k\Omega}{7.3k\Omega + 1.92k\Omega}$$

Its effect on the gain is evident when we divide the two equations

$$\frac{i_B'}{i_B} = \frac{v_i \dfrac{1}{50\Omega + \dfrac{1.92k\Omega \times 7.3k\Omega}{7.3k\Omega + 1.92k\Omega} + Z_{Ci}} \dfrac{7.3k\Omega}{7.3k\Omega + 1.92k\Omega}}{v_i \dfrac{7.3k\Omega}{7.3k\Omega + 50\Omega} \dfrac{1}{1.92k\Omega + \dfrac{50\Omega \times 7.3k\Omega}{7.3k\Omega + 50\Omega}}}$$

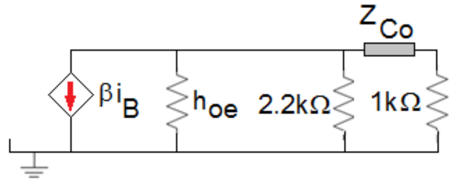
Simplifying we get: $$\frac{i_B'}{i_B} = \frac{1}{1 - \dfrac{j}{\omega C_i \left(50\Omega + \dfrac{1.92k\Omega \times 7.3k\Omega}{7.3k\Omega + 1.92k\Omega} \right)}}$$

In absolute value $$\left| \frac{i_B'}{i_B} \right| = \frac{1}{\sqrt{1 + \left(\dfrac{1}{2\pi C_i \left(50\Omega + \dfrac{1.92k\Omega \times 7.3k\Omega}{7.3k\Omega + 1.92k\Omega} \right) f} \right)^2}} = \frac{1}{\sqrt{1 + \left(\dfrac{f_{LCi}}{f} \right)^2}}$$

The cut-off frequency is:

$$f_{LCi} = \frac{1}{2\pi \times 10\mu F \times \left(50\Omega + \dfrac{1.92k\Omega \times 7.3k\Omega}{7.3k\Omega + 1.92k\Omega} \right)} = 10.1Hz$$

Similarly, the ***output coupling capacitor*** connects the signal in series with the load as shown below.



The effect of this capacitor can be seen when comparing the output voltage with and without this impedance:

$$\frac{v_o'}{v_o}=\frac{\beta i_B\times\frac{\left(2.2k\Omega//\frac{1}{h_{oe}}\right)}{2.2k\Omega//\frac{1}{h_{oe}}+1k\Omega+Z_{Coi}}\times 1k\Omega}{\beta i_B\times\frac{\left(2.2k\Omega//\frac{1}{h_{oe}}\right)}{2.2k\Omega//\frac{1}{h_{oe}}+1k\Omega}\times 1k\Omega}$$

Simplifying we get

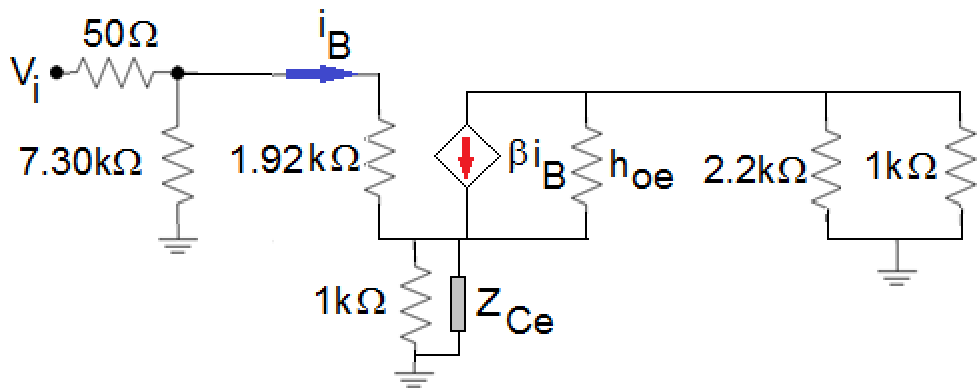
$$\frac{v_o'}{v_o}=\frac{2.2k\Omega//\frac{1}{h_{oe}}+1k\Omega}{2.2k\Omega//\frac{1}{h_{oe}}+1k\Omega+Z_{Coi}}$$

In absolute value $\left|\frac{v_o'}{v_o}\right|=\frac{1}{\sqrt{1+\frac{1}{\left(2\pi C_o\left(2.2k\Omega//\frac{1}{h_{oe}}+1k\Omega\right)f\right)^2}}}=\frac{1}{\sqrt{1+\left(\frac{f_{LCo}}{f}\right)^2}}$

Then the cut-off frequency associated with this capacitor is

$$f_{LCo}=\frac{1}{2\pi\times 1\mu F\times(2.2k\Omega//\frac{1}{12\mu S}+1k\Omega)}=50.6Hz$$

The case of the ***emitter capacitor*** is a bit different. Notice that including this capacitor the equivalent circuit is as follows:



The main effect is that the base current is smaller than if this impedance where a short circuit. We can compare the base current in the two cases:

$$i_B' = v_i \frac{7.3k\Omega}{7.3k\Omega + 50\Omega} \frac{1}{1.92k\Omega + \frac{50\Omega \times 7.3k\Omega}{7.3k\Omega + 50\Omega} + (180+1)(Z_{Ci} // 1k\Omega)}$$

Giving the following ratio

$$\frac{i_B'}{i_B} = \frac{v_i \frac{7.3k\Omega}{7.3k\Omega + 50\Omega} \frac{1}{1.92k\Omega + \frac{50\Omega \times 7.3k\Omega}{7.3k\Omega + 50\Omega} + (180+1)(Z_{Ci} // 1k\Omega)}}{v_i \frac{7.3k\Omega}{7.3k\Omega + 50\Omega} \frac{1}{1.92k\Omega + \frac{50\Omega \times 7.3k\Omega}{7.3k\Omega + 50\Omega}}}$$

Simplifying
$$\frac{i_b'}{i_b} = \frac{1}{1 + \frac{(Z_{Ci} // 1k\Omega)}{\left(1.92k\Omega + \frac{50\Omega \times 7.3k\Omega}{7.3k\Omega + 50\Omega}\right)/(180+1)}}$$

It is typical to neglect the emitter resistance when calculating this cut-off frequency, which gives us:

$$f_{LCe} = \frac{1}{2\pi \times 10\mu F \times \left(1.92k\Omega + \frac{50\Omega \times 7.3k\Omega}{7.3k\Omega + 50\Omega}\right)/(180+1)} = 1.46kHz$$

The effect of the resistance in parallel is that the attenuation factor does not decrease indefinitely when the frequency is lowered. Instead, it approaches an asymptote with a minimum impedance of 1kohm in this case.

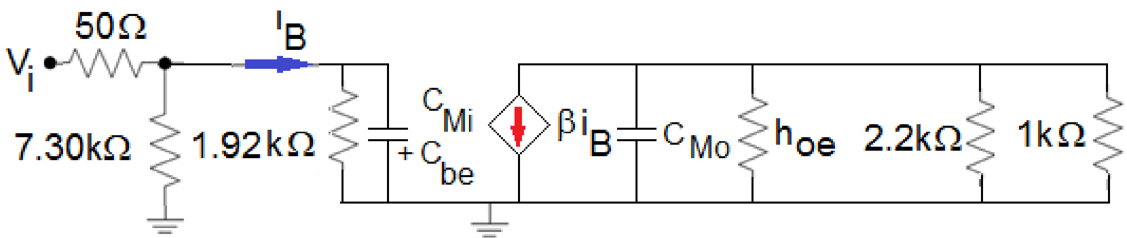
Calculation of high cut-off frequencies:

There is a 4.5pF capacitance associated with the base-collector terminals of the transistor. The corresponding Miller capacitances are:

$$C_{Mi} = 4.5pF(1+61.7) = 282pF$$

$$C_{Mo} = 4.5pF\left(1+\frac{1}{61.7}\right) = 4.57pF$$

The AC model with these capacitances is shown below:



This allows us to calculate the high cut-off frequencies.

For the ***input,***

$$f_{Hi} = \frac{1}{2\pi \times (282pF + 8pF) \times (7.3k\Omega // 1.92k\Omega // 50\Omega)} = 11.3MHz$$

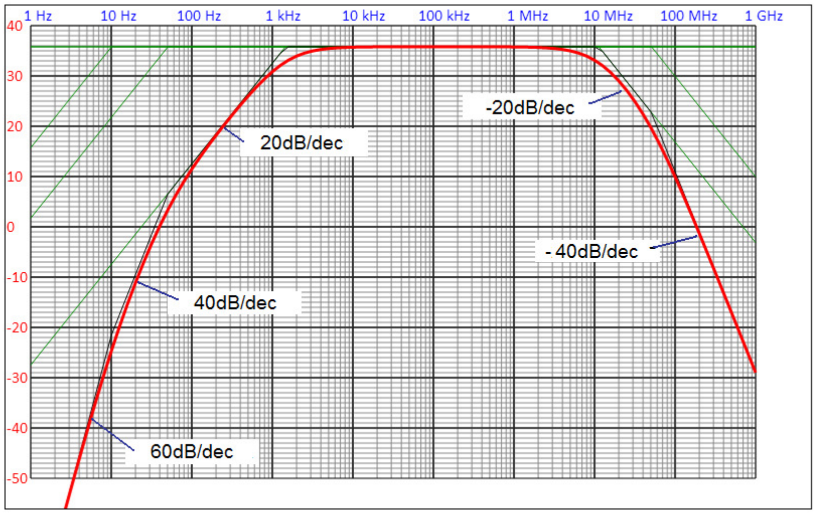
For the **output**,

$$f_{Ho} = \frac{1}{2\pi \times (4.57pF) \times \left(1k\Omega // 2.2k\Omega // \frac{1}{12\mu S}\right)} = 51MHz$$

In the Bode diagram we have then

Low frequency	$f_{LCi} = 10.1Hz$, $f_{LCo} = 50.6Hz$ and $f_{LCe} = 1.46kHz$
And high frequency	$f_{Ho} = 51MHz$ and $f_{Hi} = 11.3MHz$

The dominant values that determine the bandwidth are 1.46kHz and 11.3MHz.

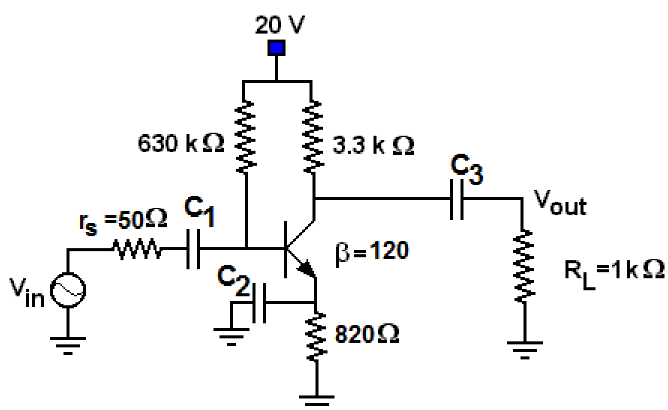


Problem 2.- In the amplifier shown below, consider that C_1=10µF, C_2=10µF and C_3=1µF.

1) Find the voltage gain with load including the source resistance. For this calculation ignore the effects of the capacitors.

2) Determine the correction factors in low frequency due to the coupling and emitter capacitors.

3) Graph the gain, including the three effects determined above.



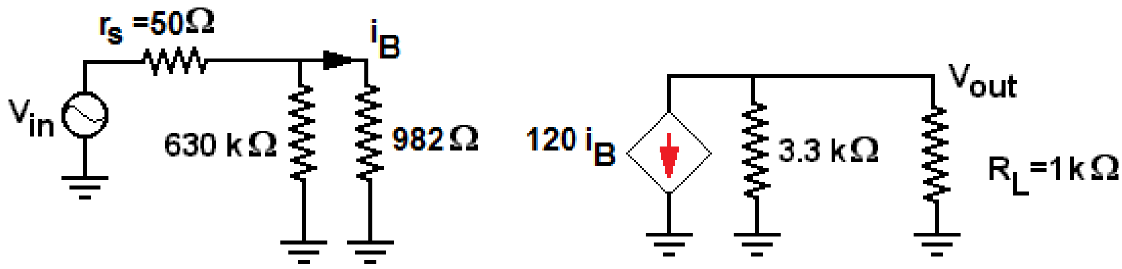
Solution: We first find the "Q" point of the transistor:

$$I_B = \frac{20V - 0.7V}{630k\Omega + 820\Omega \times 121} = 26.5\mu A$$

$$V_{CE} = 20V - 120 \times 26.5\mu A \times 3.3k\Omega - 121 \times 26.5\mu A \times 820\Omega = 6.89V$$

The dynamic base-emitter resistance is $r_d = \dfrac{26mV}{26.5\mu A} = 982\Omega$

In small signal, without considering the capacitors we get:



The base current in small signal is $$i_B = \frac{V_{in}\left(\frac{630k\Omega}{630k\Omega + 50\Omega}\right)}{982\Omega + 50\Omega\left(\frac{630k\Omega}{630k\Omega + 50\Omega}\right)}$$

The correction in parenthesis is very close to 1, so it is possible to neglect it and work with the approximation

$$i_B = \frac{V_{in}}{982\Omega + 50\Omega}$$

The output voltage is

$$V_{out} = -120\frac{V_{in}}{982\Omega + 50\Omega}\left(\frac{1k\Omega \times 3.3k\Omega}{1k\Omega + 3.3k\Omega}\right) = -89.2V_{in}$$

Which is a gain of -89.2

To find the correction factors we will analyze each capacitor separately. We start with C_1 whose effect is to insert a complex impedance in series with the input impedance calculated previously, which gives us the output voltage:

$$V_{out} = -120\frac{V_{in}}{982\Omega + 50\Omega + \frac{1}{j\omega C_1}}\left(\frac{1k\Omega \times 3.3k\Omega}{1k\Omega + 3.3k\Omega}\right)$$

This factor is isolated below

$$V_{out} = -120\frac{V_{in}}{982\Omega + 50\Omega}\left(\frac{1k\Omega \times 3.3k\Omega}{1k\Omega + 3.3k\Omega}\right)\left(\frac{982\Omega + 50\Omega}{982\Omega + 50\Omega + \frac{1}{j\omega C_1}}\right)$$

Taking absolute value:

$$\left|\frac{982\Omega+50\Omega}{982\Omega+50\Omega+\frac{1}{j\omega C_1}}\right| = \frac{1}{\sqrt{1+\left(\frac{f_{C1}}{f}\right)^2}} \rightarrow f_{C1} = \frac{1}{2\pi(1032\Omega)(10\mu F)} = 15.4Hz$$

The effect of C_3 is also straightforward. We see that it appears in the voltage output, which is given by

$$\mathrm{V_{out}} = -120\frac{\mathrm{V_{in}}}{982\Omega+50\Omega}\left(\frac{1\mathrm{k}\Omega\times 3.3\mathrm{k}\Omega}{1\mathrm{k}\Omega+\frac{1}{j\omega C_3}+3.3\mathrm{k}\Omega}\right)$$

Isolating the factor of this effect in the equation, we get:

$$\mathrm{V_{out}} = -120\frac{\mathrm{V_{in}}}{982\Omega+50\Omega}\left(\frac{1\mathrm{k}\Omega\times 3.3\mathrm{k}\Omega}{1\mathrm{k}\Omega+3.3\mathrm{k}\Omega}\right)\frac{(1\mathrm{k}\Omega+3.3\mathrm{k}\Omega)}{\left(1\mathrm{k}\Omega+\frac{1}{j\omega C_3}+3.3\mathrm{k}\Omega\right)}$$

Taking the absolute value:

$$\left|\frac{(1\mathrm{k}\Omega+3.3\mathrm{k}\Omega)}{\left(1\mathrm{k}\Omega+\frac{1}{j\omega C_3}+3.3\mathrm{k}\Omega\right)}\right| = \frac{1}{\sqrt{1+\left(\frac{f_{C3}}{f}\right)^2}} \rightarrow f_{C3} = \frac{1}{2\pi(4.3k\Omega)(1\mu F)} = 37.0Hz$$

The effect of capacitor C_2 is observed in the emitter resistance. Normally we consider it as a short circuit, but it is an impedance that appears in the equation for base current as follows:

$$\mathrm{V_{out}} = -120\frac{\mathrm{V_{in}}}{982\Omega+50\Omega+121\left(\frac{820\Omega\times\frac{1}{j\omega C_2}}{820\Omega+\frac{1}{j\omega C_2}}\right)}\left(\frac{1\mathrm{k}\Omega\times 3.3\mathrm{k}\Omega}{1\mathrm{k}\Omega+3.3\mathrm{k}\Omega}\right)$$

We isolate the correction factor

$$\frac{982\Omega + 50\Omega}{982\Omega + 50\Omega + 121\left(\dfrac{820\Omega \times \dfrac{1}{j\omega C_2}}{820\Omega + \dfrac{1}{j\omega C_2}}\right)} = \frac{1}{1 + \dfrac{121}{1032\Omega}\left(\dfrac{820\Omega}{820\Omega j\omega C_2 + 1}\right)}$$

In first approximation, ignoring the “1” in parenthesis we get:

$$\left|\frac{1}{1 + \dfrac{121}{1032\Omega}\left(\dfrac{1}{j\omega C_2}\right)}\right| = \frac{1}{\sqrt{1 + \left(\dfrac{f_{C2}}{f}\right)^2}} \rightarrow f_{C2} = \frac{121}{2\pi(1032\Omega)(10\mu F)} = 1.87kHz$$

The effect of all three factors combined can be seen in the following Bode diagram: