## Electronics

## More examples with BJTs

Problem 1.- Considering the circuit shown below,
a) Calculate the value of $R_{B}$ to bias the transistor in the middle of the active region.
b) Calculate the current, voltage and power of the transistor at the operating point.
c) Calculate the input impedance.
d) Calculate the output impedance.
e) Calculate the voltage gain with and without load.
f) Calculate the gain in power.


Solution: First, we do the DC analysis with the capacitors modeled as open circuits.


In the collector-emitter branch, we can write this equation
$12 V=3.3 k \Omega I_{C}+V_{C E}+(200 \Omega+820 \Omega) I_{E}$

Knowing that $I_{C}=\beta I_{B}$ and $I_{E}=(\beta+1) I_{B}$, we replace these variables obtaining
$12 V=3.3 k \Omega \beta+V_{C E}+(200 \Omega+820 \Omega)(\beta+1) I_{B}$

If we want the transistor in the middle of the active region, $\mathrm{V}_{\mathrm{CE}}$ has to be 6 V . Hence, we can calculate the base current using the equation above
$I_{B}=\frac{12 V-6 V}{3.3 k \Omega \beta+(200 \Omega+820 \Omega)(\beta+1)}=\frac{12 V-6 V}{3.3 k \Omega \times 100+(200 \Omega+820 \Omega)(100+1)}=13.9 \mu \mathrm{~A}$
Also the collector and emitter currents
$I_{C}=\beta I_{B}=100 \times 13.9 \mu A=1.39 \mathrm{~mA}$
$I_{E}=(\beta+1) I_{B}=101 \times 13.9 \mu A=1.40 \mathrm{~mA}$
To find the base resistor consider the circuit branch on the base side:
$12 V=R_{B} I_{B}+V_{B E}+(200 \Omega+820 \Omega) I_{E}$
The voltage $V_{B E}$ can be approximated to 0.7 V and with the calculated values of the currents, we find:

$$
R_{B}=\frac{12 \mathrm{~V}-0.7 \mathrm{~V}-(200 \Omega+820 \Omega) I_{E}}{I_{B}}=\frac{12 \mathrm{~V}-0.7 \mathrm{~V}-(200 \Omega+820 \Omega) 1.4 \mathrm{~mA}}{13.9 \mu \mathrm{~A}}=712 \mathrm{k} \Omega
$$

The power dissipated by the transistor is

$$
P=V_{C E} I_{C}+V_{B E} I_{B}=6 V \times 1.39 \mathrm{~mA}+0.7 \mathrm{~V} \times 13.9 \mu \mathrm{~A}=8.32 \mathrm{~mW}
$$

For the AC analysis, the base-emitter junction will be modeled as a dynamic resistance with value
$r_{d}=\frac{26 \mathrm{mV}}{I_{B}}=\frac{26 \mathrm{mV}}{13.9 \mu \mathrm{~A}}=1.88 \mathrm{k} \Omega$
From the point of view of the collector, the transistor behaves as a dependent current source. This gives us the following AC model.


Because the capacitors behave as short-circuits in AC, the $820 \Omega$ emitter resistance doesn't participate in the analysis, but the $200 \Omega$ does. The current that runs through this resistance is $(\beta+1)$ times the base current, so it can be modeled as a $200 \Omega \times(\beta+1)$ resistance from the point of view or the base.


The input impedance of the amplifier is
$Z_{i}=R_{B} / /\left[r_{d}+200 \Omega \times(\beta+1)\right]=712 k \Omega / /[1.88 k \Omega+200 \Omega \times 101]=21.4 k \Omega$
The output impedance is the 3.3 kohm resistance
$Z_{o}=3.3 \mathrm{k} \Omega$
To find the voltage gain we calculate the base current in AC. To do this, we find the Thevenin equivalent of the signal source, its resistance and the base resistance:
$V_{\text {Th }}=V_{\text {in }} \frac{712 k \Omega}{712 k \Omega+500 \Omega}=0.999 V_{\text {in }} \quad \quad R_{\text {Th }}=\frac{712 k \Omega \times 500 \Omega}{712 k \Omega+500 \Omega}=500 \Omega$
The base current in AC is
$i_{B}=\frac{0.999 V_{i n}}{500 \Omega+1.88 k \Omega+200 \Omega(100+1)}$
The collector current in AC is
$i_{C}=\beta i_{B}=\frac{100 \times 0.999 V_{i n}}{500 \Omega+1.88 k \Omega+200 \Omega(100+1)}$
Without load, this current will run through the $3.3 k \Omega$ producing an output voltage of
$V_{\text {out }}=-3.3 k \Omega i_{C}=-\frac{100 \times 0.999 \times 3.3 \mathrm{k} \Omega}{500 \Omega+1.88 k \Omega+200 \Omega(100+1)} V_{\text {in }}=-14.6 V_{\text {in }}$

That is a gain of -14.6
If we connect the load, the collector current will run through the combination of resistances in parallel, that is

$$
R=3.3 k \Omega / / 800 \Omega=\frac{800 \Omega \times 3.3 k \Omega}{800 \Omega+3.3 k \Omega} V_{i n}=644 \Omega
$$

And the output voltage will be

$$
V_{\text {out }}=-644 \Omega i_{C}=-\frac{100 \times 0.999 \times 644 \Omega}{500 \Omega+1.88 \mathrm{k} \Omega+200 \Omega(100+1)} V_{i n}=-2.85 V_{\text {in }}
$$

This is a gain of -2.85
The power gain without load is zero, and with load it is:

$$
\frac{P_{\text {out }}}{P_{\text {in }}}=\left(\frac{V_{\text {out }}}{V_{\text {in }}}\right)^{2} \frac{Z_{\text {in }}}{Z_{L}}=(-2.85)^{2} \frac{21.4 \mathrm{k} \Omega}{800 \Omega}=217
$$

In decibels, this is $\quad 10 \log (217)=23.4 d B$

Problem 2.- You want to connect an instrument with low input impedance ( 8 ohm ) with an analog audio signal source ( $\sim 4 \mathrm{kHz}$ ) of 400 mV amplitude that has an output impedance of 1 kohm. The voltage gain is not crucial (it can be $\sim 1$ ), but it is important to couple the impedances.


Propose a circuit based on transistors to accomplish this and calculate
(a) The voltage gain
(b) The input impedance of the amplifier

Solution.- We could use an emitter follower to approximate the impedances. Consider, for example the following circuit that you can build in the lab and with beta $=100$.


In DC the capacitors behave as open circuits and the voltage divider that feeds the base gives us the following Thevenin equivalent

$$
V t h=10 V \frac{4.7 k \Omega}{4.7 \Omega+22 k \Omega}=1.76 \mathrm{~V} \quad \quad \text { Rth }=\frac{4.7 k \Omega \times 22 k \Omega}{4.7 k \Omega+22 k \Omega}=3.87 k \Omega
$$

The base current will be

$$
I_{B Q}=\frac{1.76 \mathrm{~V}-0.7 \mathrm{~V}}{3.87 \mathrm{k} \Omega+101 \times 0.27 \mathrm{k} \Omega}=34.0 \mu \mathrm{~A}
$$

The Q point is

$$
I_{C Q}=\beta I_{B Q}=100 \times 34.0 \mu A=3.4 \mathrm{~mA}
$$

$$
V_{C E}=10 \mathrm{~V}-3.4 \mathrm{~mA} \times 270 \Omega=9.08 \mathrm{~V}
$$

The AC dynamic resistance is: $\quad r_{d}=\frac{26 m V}{34 \mu \mathrm{~A}}=765 \Omega$

The load impedance in parallel with the emitter resistance give us
$R_{E} / / R_{L}=\frac{8 \Omega \times 270 \Omega}{270 \Omega+8 \Omega}=7.77 \Omega$
Reflected to the base it is
$\beta\left(R_{E} / / R_{L}\right)=777 \Omega$
The AC model is


The input signal has a Thevenin equivalent
$v_{t h}=v_{i n} \frac{3.87 \mathrm{k} \Omega}{1 \mathrm{k} \Omega+3.87 \mathrm{k} \Omega}=0.795 v_{i n}$
$r_{t h}=\frac{3.87 \mathrm{k} \Omega \times 1 \mathrm{k} \Omega}{1 \mathrm{k} \Omega+3.87 \mathrm{k} \Omega}=0.795 \mathrm{k} \Omega$
So, the voltage gain is:

$$
A=0.795 v_{i n} \frac{777 \Omega}{777 \Omega+765 \Omega+795 \Omega}=0.264 v_{i n}
$$

And the input impedance

$$
Z_{i n}=1 k \Omega+3.87 k \Omega / /(765 \Omega+777 \Omega)=2.1 k \Omega
$$

Problem 3.- The following circuit is a typical amplifier based on one transistor. It is a simple circuit, but it has the drawback that the strong dependence of the Q point on beta.
Calculate the Q point $\left(\mathrm{V}_{\mathrm{CE}}, \mathrm{I}_{\mathrm{CQ}}\right)$ and the voltage gain for beta 50 and 150 .


Solution.- This is a direct polarization with a fixed DC base current:
$I_{B Q}=\frac{20-0.7}{630 k}=30.6 \mu \mathrm{~A}$
The dynamic resistance is $r_{d}=\frac{26 m V}{30.6 \mu A}=849 \Omega$
The collector current and collector-emitter voltage depend on beta:
$\beta=50 \rightarrow I_{C Q}=50 \times 30.6 \mu A=1.53 \mathrm{~mA} \rightarrow V_{C E}=20-1.53 \mathrm{~mA} \times 3.3 \mathrm{k} \Omega=14.9 \mathrm{~V}$
$\beta=150 \rightarrow I_{C Q}=150 \times 30.6 \mu \mathrm{~A}=4.59 \mathrm{~mA} \rightarrow V_{C E}=20-4.59 \mathrm{~mA} \times 3.3 \mathrm{k} \Omega=4.83 \mathrm{~V}$
The two voltage gains are:
$\beta=50 \rightarrow A=-50 \times \frac{1 k / / 3.3 k}{849 \Omega}=-45.2$
$\beta=150 \rightarrow A=-150 \times \frac{1 k / / 3.3 k}{849 \Omega}=-135.6$

