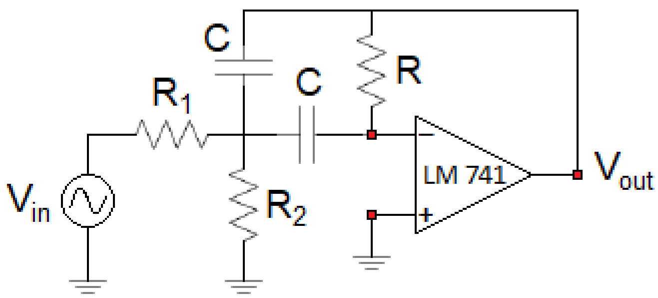
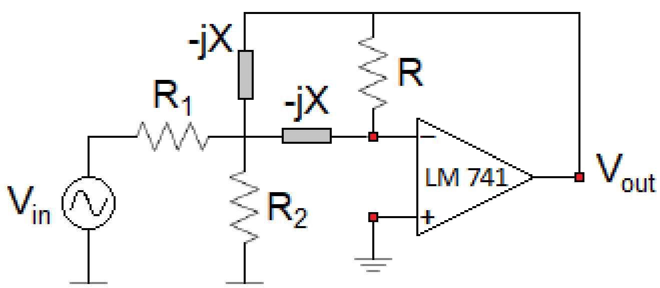
Electronics

Special filters

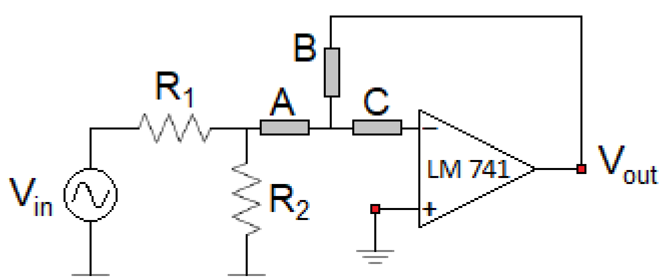
Problem 1.-The circuit uses only one opamp, but with three resistors and two capacitors as shown in the schematic behaves as a special filter. Calculate its gain.



Solution: We can define $X = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ as the impedance of the capacitor and then the circuit using complex numbers to model the circuit:



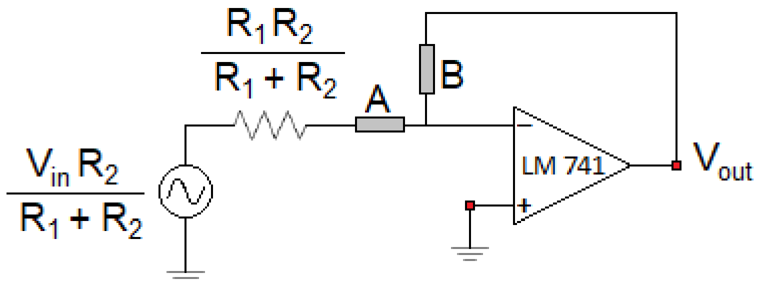
Notice that the two capacitors and the resistance R form a delta configuration, which we can transform into a star as shown below.



The values of the impedances A, B and C are given by the usual transformations:

$$A = \frac{(-jX)(-jX)}{-jX - jX + R} \qquad B = \frac{(R)(-jX)}{-jX - jX + R} \qquad C = \frac{(R)(-jX)}{-jX - jX + R}$$

Notice that impedance C is irrelevant because the current entering the opamp is negligible. Notice also that the signal source and resistors R_1 and R_2 can be replaced by a Thevenin equivalent, so the circuit becomes:



You can recognize that this circuit behaves as an inverting amplifier with gain:

$$G = \frac{-B}{A + \frac{R_1 R_2}{R_1 + R_2}} = \frac{-\frac{(R)(-jX)}{-jX - jX + R}}{\frac{(-jX)(-jX)}{-jX - jX + R} + \frac{R_1 R_2}{R_1 + R_2}} = \frac{1}{-\frac{2R_1 R_2}{R(R_1 + R_2)} + j\left(\frac{X}{R} - \frac{R_1 R_2}{X(R_1 + R_2)}\right)}$$

Its absolute value is: $|G| = \frac{1}{\sqrt{\left(\frac{2R_1 R_2}{R(R_1 + R_2)}\right)^2 + \left(\frac{X}{R} - \frac{R_1 R_2}{(R_1 + R_2)X}\right)^2}}$

And including the factor $\frac{R_2}{R_1 + R_2}$ from the Thevenin transformation, we get:

$$|G| = \frac{1}{\sqrt{\left(\frac{2R_1}{R}\right)^2 + \left(X\frac{(R_1 + R_2)}{R_2 R} - \frac{R_1}{X}\right)^2}}$$

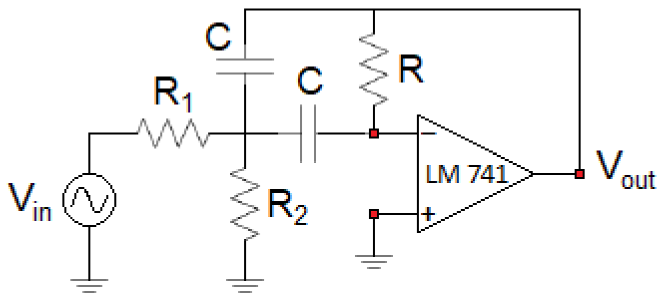
The maximum value of the gain happens when the second term under the square root is zero:

$$\frac{X}{R} - \frac{R_1 R_2}{(R_1 + R_2)X} = 0 \rightarrow X_{MAX} = \sqrt{\frac{R R_1 R_2}{R_1 + R_2}} \rightarrow f_{MAX} = \frac{1}{2\pi C}\sqrt{\frac{R_1 + R_2}{R R_1 R_2}}$$

At that point it will be: $G_{MAX} = \frac{R}{2R_1}$

The gain can also be written as: $$G = \frac{R}{2R_1}\frac{1}{\sqrt{1 + \left(\frac{R}{2X_{MAX}}\right)^2\left(\frac{f_{MAX}}{f} - \frac{f}{f_{MAX}}\right)^2}}$$

Problem 2.- Choose components for the following circuit to filter a signal in such a way that it has a peak gain of 0dB at 4.4kHz



Solution: You can demonstrate (see problem above) that the gain is

$$|G| = \frac{R}{2R_1} \frac{1}{\sqrt{1 + \left(\frac{R(R_1 + R_2)}{4R_1R_2}\right)\left(\frac{f_{MAX}}{f} - \frac{f}{f_{MAX}}\right)^2}}$$

Where the maximum value is: $G_{MAX} = \frac{R}{2R_1}$

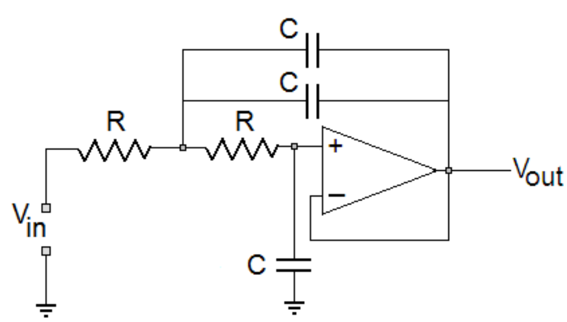
At a frequency $f_{MAX} = \frac{1}{2\pi C}\sqrt{\frac{R_1 + R_2}{RR_1R_2}}$

To have a peak of 0dB we need $R = 2R_1$, for example R = 2kΩ and R_1 = 1kΩ.
For that to happen at 4.4kHz, we need

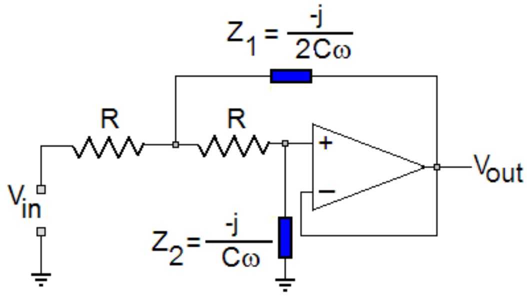
$$4400Hz = \frac{1}{2\pi C}\sqrt{\frac{1k\Omega + R_2}{2k\Omega \times 1k\Omega \times R_2}}$$

For example R_2 = 1kΩ and C = 36.2nF

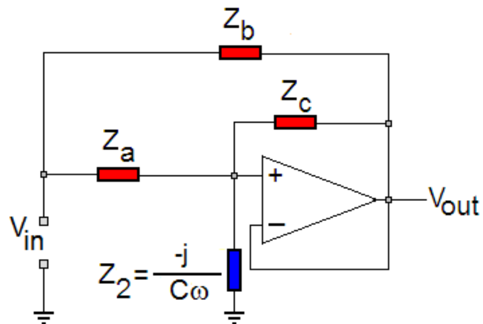
Problem 3.- In the following filter calculate the gain as a function of frequency.



Solution: The circuit, with the capacitors replaced by complex impedances, looks like this:



To simplify the circuit, we replace the star connection made by the two resistors and Z_1 for a delta connection as follows:



The value of Z_a is given by:

$$Z_a = R + R + \frac{R^2}{-\frac{j}{2C\omega}} = 2R + 2R^2 Cj\omega$$

We can ignore Z_b since it is connected between the source and the opamp output and we can also ignore Z_c because no current will flow through it. Then the filter behaves like a voltage divider, so the gain is:

$$G = \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_2 + Z_a} = \frac{-\frac{j}{C\omega}}{2R + 2R^2 Cj\omega - \frac{j}{C\omega}}$$

The absolute value of the gain is:

$$|G| = \frac{\frac{1}{C\omega}}{\sqrt{4R^2 + \left(2R^2 C\omega - \frac{1}{C\omega}\right)^2}}$$

This can be simplified to give: $$|G| = \frac{1}{\sqrt{1 + 4R^4 C^4 \omega^4}}$$

We can define the cutoff frequency with the equation:

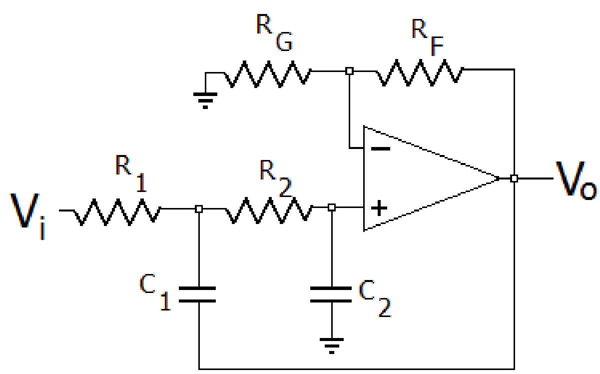
$$\omega_\circ = \frac{1}{\sqrt{2}RC} \rightarrow f_\circ = \frac{1}{2\pi\sqrt{2}RC}$$

And the gain can be then written as $|G| = \dfrac{1}{\sqrt{1+\left(\dfrac{f}{f_\circ}\right)^4}}$

Here, after the cutoff frequency the drop in gain is 40 dB per decade, which is twice what you get with a simple RC filter.

Problem 4.- The following low-pass filter is a compact design with two poles that in its mid band has a gain $A = 1 + \dfrac{R_F}{R_G}$

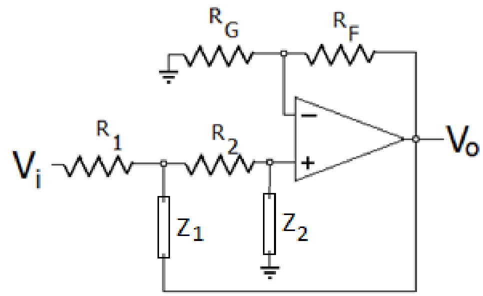
And cut-off frequency $f_C = \dfrac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$



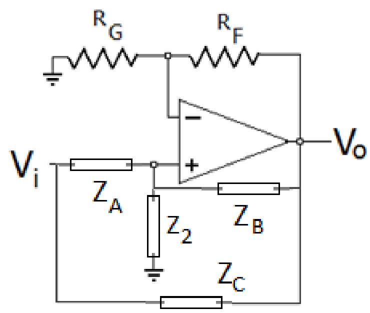
Its Bode diagram can be modeled as two straight lines that intersect at the cut-off frequency, however its precise form depends on the specific component values.
You are asked to find the precise curve close to the cut-off frequency.

Solution: Replacing the capacitors by complex impedances

$$Z_1 = -\frac{j}{\omega C_1} \text{ and } Z_2 = -\frac{j}{\omega C_2}$$



The star circuit formed by R_1, R_2 and Z_1 can be converted into a delta



With the values

$$Z_A = R_1 + R_2 + jR_1R_2\omega C_1$$

$$Z_B = Z_1 + R_2 + \frac{Z_1R_2}{R_1} = R_2 - \frac{j}{\omega C_1}\left(1+\frac{R_2}{R_1}\right)$$

$$Z_C = Z_1 + R_1 + \frac{Z_1R_1}{R_2} = R_1 - \frac{j}{\omega C_1}\left(1+\frac{R_1}{R_2}\right)$$

We notice that Z_C does not participate in the calculations. The branch formed by R_G and R_F behaves as a voltage divider with its voltage going to the opamp's inverting input.

$$V_- = V_o\frac{R_G}{R_G + R_F}$$

We can find V_i as a function of V_o and then invert the equation to find the gain. With this plan, consider:

Current through Z_2 $$I_{Z2} = V_o\frac{R_G}{Z_2(R_G + R_F)}$$

Current through Z_B $$I_{ZB} = -V_o\frac{R_F}{Z_B(R_G + R_F)}$$

Current through Z_A $$I_{ZA} = V_o\frac{R_G}{Z_2(R_G + R_F)} - V_o\frac{R_F}{Z_B(R_G + R_F)}$$

Voltage V_i
$$V_i = V_o \frac{R_G}{R_G + R_F} + V_o \frac{R_G Z_A}{Z_2(R_G + R_F)} - V_o \frac{R_F Z_A}{Z_B(R_G + R_F)}$$

Inverting the equation

$$\frac{V_o}{V_i} = \frac{1}{\frac{R_G}{R_G + R_F} + \frac{R_G Z_A}{Z_2(R_G + R_F)} - \frac{R_F Z_A}{Z_B(R_G + R_F)}}$$

Simplifying

$$\frac{V_o}{V_i} = \frac{R_G + R_F}{R_G + \frac{R_G Z_A}{Z_2} - \frac{R_F Z_A}{Z_B}}$$

Next, you can put this equation in your favorite numerical program and graph the gain close to the cut-off frequency, like in this example: