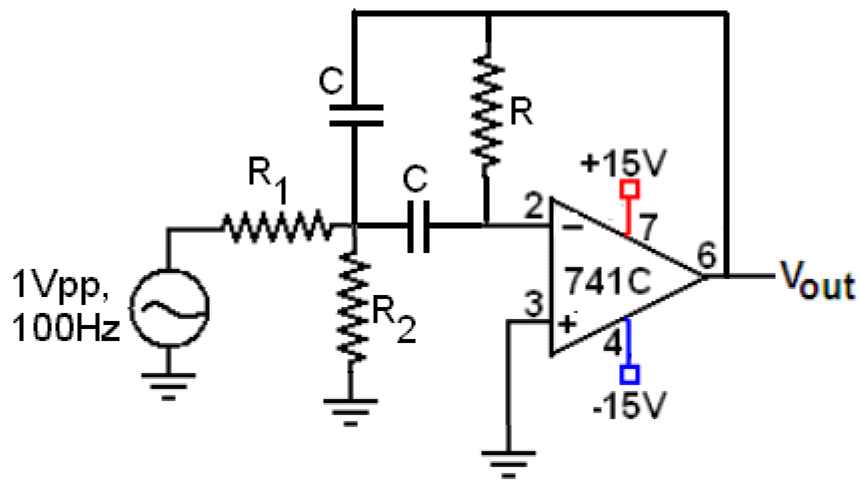
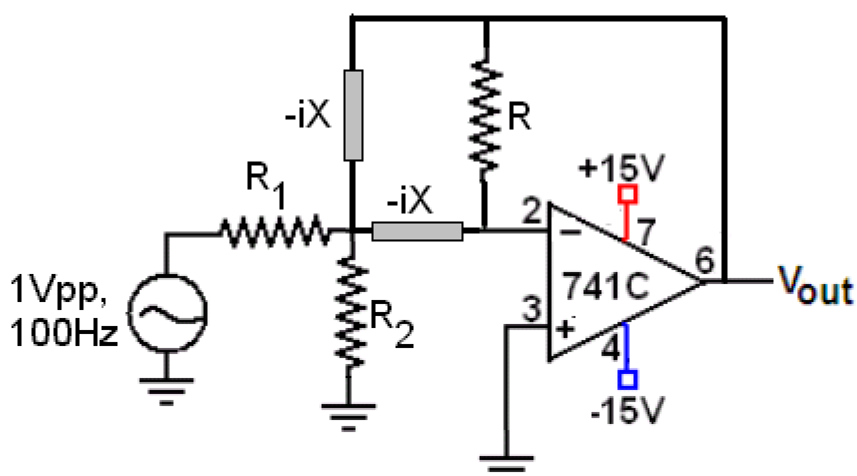
Electronics Lab

Filter analysis

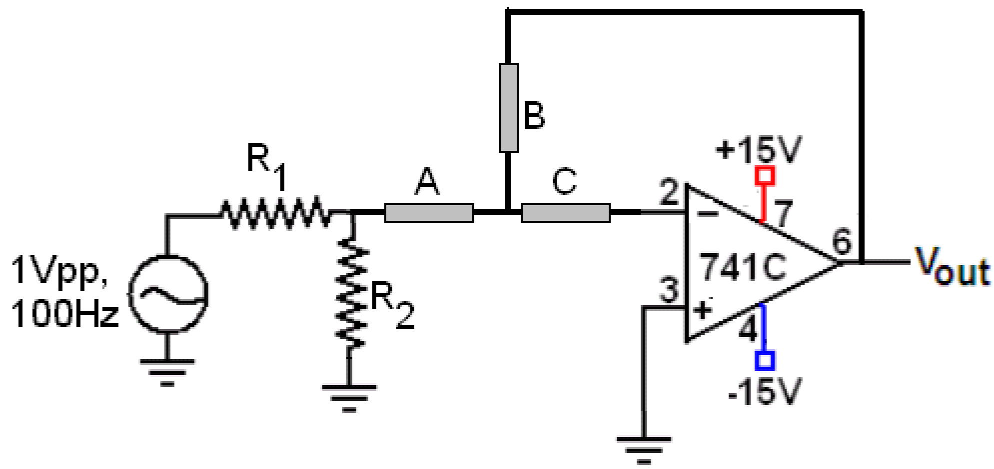
***Active narrow band-pass filter*.**
The circuit uses only one opamp, but there are three resistors and two capacitors as shown in the schematic:



We can define $X = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ as the impedance of the capacitor and then the circuit using complex impedances will look as follows:



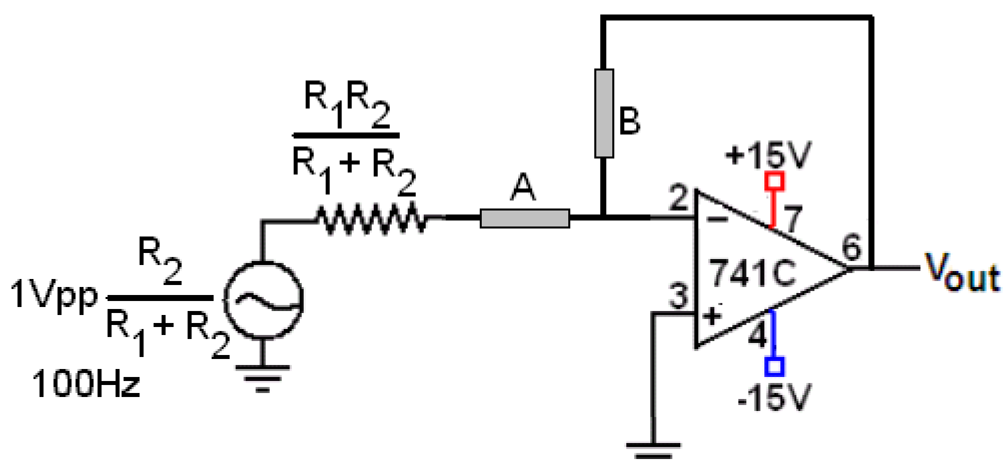
Now, notice that the two capacitors and the resistance "R" form a delta configuration, which we can transform into a star as follows:



The values of the impedances A, B and C are given by the usual transformations:

$$A = \frac{(-iX)(-iX)}{-iX - iX + R} \qquad B = \frac{(R)(-iX)}{-iX - iX + R} \qquad C = \frac{(R)(-iX)}{-iX - iX + R}$$

But notice that the impedance C is irrelevant because the current entering the opamp is negligible, so it won't play a role. Another point: Notice that the signal and the two resistors R_1 and R_2 can be replaced by a Thevenin equivalent, so the circuit becomes:



You can tell that this circuit behaves as an inverting amplifier with gain:

$$Gain_{INV.AMP} = \frac{-B}{A + \frac{R_1 R_2}{R_1 + R_2}} = \frac{-\frac{(R)(-iX)}{-iX - iX + R}}{\frac{(-iX)(-iX)}{-iX - iX + R} + \frac{R_1 R_2}{R_1 + R_2}} = \frac{1}{-\frac{2R_1 R_2}{R(R_1 + R_2)} + i\left(\frac{X}{R} - \frac{R_1 R_2}{X(R_1 + R_2)}\right)}$$

The absolute value of the gain is: $\left|Gain_{INV.AMP}\right| = \dfrac{1}{\sqrt{\left(\dfrac{2R_1R_2}{R(R_1+R_2)}\right)^2 + \left(\dfrac{X}{R} - \dfrac{R_1R_2}{(R_1+R_2)X}\right)^2}}$

The overall gain will need to consider the Thevenin voltage, so it has a factor of $\dfrac{R_2}{R_1+R_2}$, giving:

$$\left|Gain\right| = \frac{1}{\sqrt{\left(\dfrac{2R_1}{R}\right)^2 + \left(X\dfrac{(R_1+R_2)}{R_2R} - \dfrac{R_1}{X}\right)^2}}$$

The maximum value of the gain happens when the second term under the square root is zero:

$$\frac{X}{R} - \frac{R_1R_2}{(R_1+R_2)X} = 0 \rightarrow X_{MAX} = \sqrt{\frac{RR_1R_2}{R_1+R_2}} \rightarrow f_{MAX} = \frac{1}{2\pi C}\sqrt{\frac{R_1+R_2}{RR_1R_2}}$$

At that point the gain will be: $Gain_{MAX} = \dfrac{R}{2R_1}$

And we can write the gain also as:

$$\left|Gain\right| = \frac{1}{\sqrt{\left(\dfrac{2R_1}{R}\right)^2 + \left(\dfrac{R_1}{X_{MAX}}\right)^2\left(\dfrac{f_{MAX}}{f} - \dfrac{f}{f_{MAX}}\right)^2}}$$