Thermal Physics

Thermodynamic Cycles

Problem 1.- A sports car has an engine with the following specs:

6,496 cc 6.5 liters V-12 engine with 88 mm bore, 89 mm stroke, 11 compression ratio, double overhead cam, variable valve timing/camshaft and four valves per cylinder.
Premium unleaded fuel 91 grade.
Multi-point injection fuel system, 26.4 gallon, main premium unleaded fuel tank.
Power: 477 kW, 640 HP SAE @ 8,000 rpm; 660 ft lb, 895 Nm @ 6,000 rpm

Calculate the thermal efficiency of this car and the minimum amount of heat generated per second to deliver its peak power of 640 HP.

Solution:

We can calculate the efficiency of the engine:

$$\eta = 1 - \frac{1}{r_v^{\gamma-1}} = 1 - \frac{1}{(11)^{0.4}} = 0.617$$

This allows us to find the heat necessary per second:

$$\eta = \frac{W}{Q} \rightarrow Q = \frac{W}{\eta} = \frac{477\text{kW}}{0.617} = \mathbf{773\ kW}$$

Problem 2.- A modern diesel engine delivers up to 350 HP, up to 860 lb-ft torque and displaces 7.2L. It has a compression ratio of 18:1. Estimate the maximum temperature reached by the mixture after ignition if the cutoff ratio is 1.95

[For this estimation assume that air behaves like an ideal gas with gamma = 1.4]

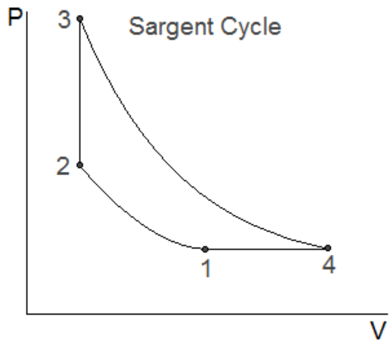
Solution: The adiabatic compression ends with air at a temperature T_2 that we can calculate from the adiabatic equation:

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \rightarrow T_2 = \frac{T_1 V_1^{\gamma-1}}{V_2^{\gamma-1}} = T_1 r_v^{\gamma-1} = 300\text{K}(18)^{1.4-1} = 953.3\text{K}$$

After burning the fuel, the mixture reaches a temperature T_3 that we can calculate from the ideal gas law. Taking into account that it is a constant pressure process:

$$\frac{T_2}{V_2} = \frac{T_3}{V_3} \rightarrow T_3 = \frac{T_2}{V_2} V_3 = T_2 r_c = 953.3\text{K}(1.95) = \mathbf{1{,}858\ K}$$

Problem 3.- The diagram below is an approximation to a Sargent cycle run on an ideal gas. A constant pressure path (4-1) and a constant volume path (2-3) are connected by two adiabatic paths (1-2 and 3-4). Assume that the heat capacities, C_P and C_V are known.
Find the thermal efficiency of the cycle.



Solution: The heat is delivered to the gas in the process 2-3 which is at constant volume, so:

$$Q_{23}=C_V(T_3\text{-}T_2)$$

The heat dump occurs in the process 4-1 and is at constant pressure, so:

$$Q_{41}=C_P(T_4\text{-}T_1)$$

With this choice of sign, the heat is positive and the work is given by: $W = Q_{23} - Q_{41}$ so the efficiency is:

$$\eta=\frac{W}{Q_{23}}=\frac{Q_{23}\text{-}Q_{41}}{Q_{23}}=1\text{-}\frac{Q_{41}}{Q_{23}}=1\text{-}\frac{C_P(T_4\text{-}T_1)}{C_V(T_3\text{-}T_2)}$$

Problem 4.- What would be the compression ratio of an Otto engine whose thermal efficiency is the same as a diesel engine that has a compression ratio of 19:1 and a cutoff ratio of 2.1
[For this problem assume that air behaves like an ideal gas with gamma=1.4]

Solution: The efficiency of the Diesel engine is:

$$\eta = 1-\frac{1}{\gamma}\frac{1}{r_v^{\gamma-1}}\left(\frac{r_c^{\gamma}-1}{r_c-1}\right)=1-\frac{1}{1.4}\left(\frac{1}{19^{0.4}}\right)\left(\frac{2.1^{1.4}-1}{2.1-1}\right)=0.6349$$

To have the same efficiency with an Otto cycle:

$$0.6349 = 1-\frac{1}{r_V^{\gamma-1}} \rightarrow 1-0.6349=\frac{1}{r_V^{\gamma-1}} \rightarrow r_V = \sqrt[\gamma-1]{\frac{1}{1-0.6349}} = \sqrt[0.4]{\frac{1}{1-0.6349}} = \mathbf{12.42}$$

Problem 5.- Calculate the efficiency of an Otto cycle that takes air at 273K and 0.9 atm and has a compression ratio of 9 to 1.
For calculation purposes, take the initial volume as 5.5 liters.
Draw the cycle and calculate the intermediate values.

Solution: We could directly use a formula to find the efficiency without calculating intermediate values, but this exercise is proposed to gain insight in how the Otto cycle works, so we will calculate intermediate points.

The number of moles in the cycle is given by

$$n = \frac{PV}{RT} = \frac{0.9 \times 1.013 \times 10^5 \times 0.0055}{8.314 \times 273} = 0.221$$

Consider the compression stroke: Here air will be compressed adiabatically from an initial volume of 5.5 liters to a final volume of 5.5/9 liters. This adiabatic process is shown in the figure as the black curve from 1 to 2.

During this compression the pressure will increase from 0.9atm=0.9×1.03×10^5 pascals following the equation

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \rightarrow P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 0.9 \times 1.013 \times 10^5 (9)^{1.4} = 1.976 \times 10^6 \ pascals$$

We can also calculate the final temperature

$$TV_1^{\gamma-1} = T_2 V_2^{\gamma-1} \rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 273(9)^{1.4-1} = 657K$$

The next step is the explosion of the mixture air and gasoline, which happens at constant volume. Since we have the final temperature (1,100K), we can calculate the heat delivered by burning the fuel:

$$Q_{in} = \frac{5}{2} \mathrm{nR\Delta T} = \frac{5}{2} 0.221 \times 8.314(1100 - 657) = 2{,}035J$$

This increase in temperature will also increase the pressure to a value given by

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \rightarrow P_3 = P_2 \left(\frac{T_3}{T_2}\right) = 1.976 \times 10^6 \left(\frac{1{,}100}{657}\right) = 3.308 \times 10^6 \ pascals$$

The next process in the cycle is the second adiabatic process, but this is an expansion, where the hot gas will produce work. The blue curve from 3 to 4 in the figure represents this process.

The final temperature can be calculated from the relation

$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \rightarrow T_4 = T_3 \left(\frac{V_3}{V_2}\right)^{\gamma-1} = 1{,}100 \left(\frac{1}{9}\right)^{1.4-1} = 457K$$

Finally, the cycle will go back to the initial condition while emitting heat as follows

$$Q_{out} = \frac{5}{2}\text{nR}\Delta\text{T} = \frac{5}{2}0.221\times 8.314(457-273) = 845J$$

The efficiency is

$$\eta = \frac{Work}{Q_{in}} = \frac{Q_{in}-Q_{out}}{Q_{in}} = \frac{2035-845}{2035} = 0.58 = 58\%$$