Thermal Physics

Thermodynamic Processes

Problem 1.- Calculate the work delivered by the isothermal expansion of 5 kg of air at T=600 K from an initial pressure of p_i = 8 atm to a final pressure of p_f = 2 atm.

Solution: The number of moles can be approximated by:

$$n = \frac{5000\text{g}}{29\text{g/mole}} = 172.4\text{moles}$$

The ratio of volumes (final volume/initial volume) is exactly the inverse of the ratio of pressures (initial pressure/final pressure) because this is an isothermal process, so:

$$W = \int_{V_{initial}}^{V_{final}} pdV = nRT\ln\frac{V_{final}}{V_{initial}} = (8.314\text{J/K})(600\text{K})\ln(4) = \mathbf{1.18\times10^{6}\ J}$$

Problem 2.- Calculate the work done by an ideal gas expanding adiabatically from the state (V_i, P_i) to the final state (V_f, P_f). Assume the value of gamma is known.

Solution: In an adiabatic process $pV^{\gamma} = p_{final}V_{final}^{\gamma} \rightarrow p = \frac{p_{final}V_{final}^{\gamma}}{V^{\gamma}}$, so the work done is:

$$W = \int_{V_{initial}}^{V_{final}} pdV = \int_{V_{initial}}^{V_{final}} \frac{p_{final}V_{final}^{\gamma}}{V^{\gamma}}dV = p_{final}V_{final}^{\gamma}\int_{V_{initial}}^{V_{final}} \frac{dV}{V^{\gamma}} = p_{final}V_{final}^{\gamma}\frac{\left(V_{final}^{1-\gamma} - V_{initial}^{1-\gamma}\right)}{1-\gamma}$$

$$W = \frac{p_{final}V_{final} - p_{initial}V_{initial}}{1-\gamma}$$

Problem 3.- Calculate the final temperature of a sample of air that is compressed adiabatically from initially STP conditions to a final pressure of 10.5 atm.

Solution: STP conditions means:

$$P_1 = 1.013\times10^{5}\,Pa \qquad T_1 = 273.15\text{K}$$

Since the process is adiabatic $P_1V_1{}^{\gamma} = P_2V_2{}^{\gamma}$ and since air can be considered an ideal gas (approximately): $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

To eliminate the volume in these equations we can raise the second equation to gamma and divide one equation by the other:

$$\frac{P_1^{\gamma}V_1^{\gamma}}{T_1^{\gamma}}=\frac{P_2^{\gamma}V_2^{\gamma}}{T_2^{\gamma}}\rightarrow\frac{P_1^{\gamma}V_1^{\gamma}}{T_1^{\gamma}P_1V_1^{\gamma}}=\frac{P_2^{\gamma}V_2^{\gamma}}{T_2^{\gamma}P_2V_2^{\gamma}}\rightarrow\frac{P_1^{\gamma}}{T_1^{\gamma}P_1}=\frac{P_2^{\gamma}}{T_2^{\gamma}P_2}\rightarrow T_2=T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

In our problem:

$$T_2=T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}=273.15\text{K}\left(\frac{10.5\text{atm}}{1\text{atm}}\right)^{\frac{1.4-1}{1.4}}=\mathbf{534.8\ K}$$

Problem 4.- Calculate the work delivered by the isentropic expansion of 5 kg of air from an initial condition of T=600 K and pressure of p_i = 8 atm to a final pressure of p_f = 2 atm.

Solution: We can calculate the final temperature using the result from the previous problem:

$$T_2=T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}=600\text{K}\left(\frac{2\text{atm}}{8\text{atm}}\right)^{\frac{1.4-1}{1.4}}=403.8\text{ K}$$

And the work is equal to the change in internal energy in an isentropic process:

$$W=-\Delta U=-C_v(403.8\text{K}-600\text{K})$$

To find the heat capacity notice that n=$\frac{5000\text{g}}{29\text{g/mole}}=172.4$ moles and the heat capacity at constant volume of air is 5/2R per mole, so:

$$W=-\Delta U=-5/2(8.314\text{J/K})(172.4)(403.8\text{K}-600\text{K})=\mathbf{703.1\ kJ}$$

Problem 5.- An adiabatic process involves a gas that can be approximated as ideal. Which of the following expressions is constant in such a process?

(A) TV
(B) TV^{γ}
(C) $TV^{\gamma-1}$
(D) $T^{\gamma}V$
(E) $T^{\gamma}V^{-1}$

Solution: Since PV^{γ}=constant in an adiabatic process and $\frac{PV}{T}$=constant for an ideal gas we can divide one equation by the other and find

$$\frac{PV^{\gamma}}{\frac{PV}{T}}=\text{constant} \rightarrow TV^{\gamma-1}=\text{constant}$$

Problem 6.- An ideal gas expands from volume V_A and pressure P_A to a volume V_B and pressure P_B in an adiabatic process. Which of the following expressions is the work done in the process?

(A) 0
(B) $nRT\ln\left(\frac{V_B}{V_A}\right)$
(C) $\frac{P_AV_A-P_BV_B}{\gamma-1}$
(D) $P_AV_A-P_BV_B$
(E) $P_A(V_B-V_A)$

Solution: (C)

Problem 7.- An ideal mono atomic gas expands from volume $V_A=1L$ and pressure $P_A=1atm$ to a volume $V_B=4L$ in an adiabatic process. Then the gas is heated at constant volume to reach a final pressure $P_C=1atm$. Considering the temperatures at points A, B and C, which of the following is true?

(A) $T_A=T_B=T_C$
(B) $T_A>T_B=T_C$
(C) $T_A>T_B>T_C$
(D) $T_C>T_A>T_B$
(E) $T_C>T_A=T_B$

Solution: (D)

Problem 8.- Calculate the work delivered by the adiabatic expansion of 1 kg of air initially at T=600 K from an initial pressure of p_i = 8 atm to a final pressure of p_f = 2 atm. Take air as an ideal gas with $\gamma=1.4$ and molecular mass $M=29$

Solution:

The number of moles is $n=\frac{1000g}{29g/mol}=34.5$

Then the initial volume is

$$V=\frac{nRT}{p}=\frac{34.5\times8.314\times600}{8\times1.013\times10^5}=0.2122m^3$$

And the final volume is

$$V_B = V_A \left(\frac{p_A}{p_B} \right)^{1/\gamma} = 0.2122 \left(\frac{8}{2} \right)^{1/1.4} = 0.5714 m^3$$

The work is

$$W = \frac{P_A V_A - P_B V_B}{\gamma - 1} = \frac{8 \times 1.013 \times 10^5 \times 0.2122 - 2 \times 1.013 \times 10^5 \times 0.5714}{1.4 - 1} = \mathbf{140\ kJ}$$