Thermal Physics

Boltzmann factors

Problem 1.- A particle can only occupy three possible states with energies $E_1= E_2=0$ and $E_3=E>0$
At temperature T, find the probability of finding the particle in state 3.

Solution: The Boltzmann factors are

$$e^{E_1/k_BT} = e^{0/k_BT} = 1$$
$$e^{E_2/k_BT} = e^{0/k_BT} = 1$$
$$e^{-E_3/k_BT} = e^{-E/k_BT}$$

Therefore, the probability of being in the third state is

$$P = \frac{e^{-E/k_BT}}{2+e^{-E/k_BT}} = \frac{1}{1+2e^{E/k_BT}}$$

Problem 2.- An impurity atom in a crystal can be approximated as a three-dimensional harmonic oscillator in thermal equilibrium with the rest of the crystal at a temperature T. If the unit excitation $\hbar\omega$ is much smaller than k_BT, the average total energy of the oscillator is

(A) $\frac{1}{2}k_BT$
(B) k_BT
(C) $\frac{3}{2}k_BT$
(D) $3k_BT$
(E) $6k_BT$

Solution: (D)

Problem 3.- A thermal system can exist in three states with energies $0, \varepsilon \text{ and } 2\varepsilon$. If each state has degeneracy 1, what is the partition function?

(A) $e^{-k_BT/\varepsilon}$
(B) $e^{-\varepsilon/k_BT}$
(C) $0+e^{-\varepsilon/k_BT}+2e^{-2\varepsilon/k_BT}$
(D) $1+e^{-\varepsilon/k_BT}+e^{-2\varepsilon/k_BT}$
(E) $e^{-3\varepsilon/k_BT}$

Solution: (D)

Problem 4.- Given a thermal system with states i and energies E_i, which of the following expressions represents the probability of finding the system in one particular state j?

(A) $\dfrac{1}{\sum e^{-E_i/k_BT}}$

(B) $\sum e^{-E_i/k_BT}$

(C) $\dfrac{e^{-E_j/k_BT}}{\sum e^{-E_i/k_BT}}$

(D) e^{-E_j/k_BT}

(E) $e^{-E_j/k_BT}\left(\sum e^{-E_i/k_BT}\right)$

Solution: (C)

Problem 5.- A certain atom can exist in a double degenerate ground state with energy 0eV and a quadruple degenerate state with energy 0.009eV. For a temperature corresponding to $k_BT = 0.09\text{eV}$ what is the ratio of probabilities of finding the atom in the excited state divided by the ground state?

(A) $4e^{10}$

(B) $2e^{0.1}$

(C) 0.5

(D) $2e^{-0.1}$

(E) $2e^{-10}$

Solution: (E)

Problem 6.- A system of is made of N independent particles that have only 3 possible states with energies $0, \varepsilon \text{ and } 2\varepsilon$. If k_BT is much larger than ε, what is the average energy of the system?

(A) 0

(B) ∞

(C) ε

(D) $N\varepsilon$

(E) $2N\varepsilon$

Solution: (D)