Thermal Physics

Discrete Random Variable

Problem 1.- In a certain quantum mechanical system the x component of the angular momentum, L_x, is quantized and can take on only the three values $-\hbar$, 0 or $\hbar$. For a given state of the system it is known that $< L_x > = 1/3\ \hbar$ and $< L_x^2 > = 2/3\ \hbar^2$

Find the probability for the x component of the angular momentum, $P(L_x)$. Sketch the result.

Solution: Let us call the probabilities of having the x component of the angular momentum, L_x, equal to $-\hbar$, 0 or $\hbar$ P1, P2 and P3.
The sum of these probabilities has to be equal to 1, so:

P1+P2+P3=1

We also know that:

$< L_x > = 1/3\ \hbar$ so $-\hbar P1+0P2+\hbar P3 = 1/3\hbar$, so $-P1+P3=1/3$

and $< L_x^2 > = 2/3\ \hbar^2$ so $\hbar^2 P1+0P2+\hbar^2 P3=2/3\hbar^2$, so P1+P3=2/3

With these three equations, we get:

P1 = 1/6 P2 = 1/3 P3 = 1/2

A sketch of the result: