Thermal Physics

Ideal gases

Problem 1.- Calculate the RMS speed of nitrogen molecules N_2 of mass m = 28u in a dilute gas (approximate as an ideal gas) at a temperature T = 1,500K.

Solution: The average kinetic energy of a molecule in an ideal gas is $\frac{3}{2}k_B T$. This does not take into account rotational or vibrational kinetic energy, only translational. So the average speed (RMS) is found with the equation:

$$\frac{1}{2}mv^2 = \frac{3}{2}k_B T \rightarrow v = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3\times 1.38\times 10^{-23}\times 1500}{28\times 1.66\times 10^{-27}}} = \mathbf{1{,}160\ m/s}$$

Problem 2.- What volume would 105.0 g of NO_2 gas occupy at 31.5 atm and 27.0 °C?

Solution: *If* NO_2 behaved like an ideal gas, its volume could be calculated as follows:

Molecular mass = 14u + 2×16u = 46u

Number of moles, $n = \frac{105g}{46g/mole} = 2.28$ moles

Temperature, T = 273 + 27 = 300 K

Pressure, p=31.5 atm = $31.5\text{atm}\left(\frac{1.013\times 10^5\text{Pa}}{1\text{atm}}\right) = 3.19\times 10^6\text{Pa}$

So:

$$V = \frac{nRT}{p} = \frac{2.28\text{moles}(8.314\text{J/Kmole})(300\text{K})}{3.19\times 10^6\text{Pa}} = \mathbf{0.00178\ m^3}$$

However, NO_2 is not in the gas phase at the conditions given in the problem, so you cannot use the ideal gas equations to calculate its volume.

Problem 3.- Calculate the molecular weight of a gas if its mass is 79.59 g, it is stored in a tank with volume 7.50 L and exerts a pressure of 60.0 atm at a constant temperature of 35.5 °C

Solution: We can calculate the number of moles using the ideal gas law:

$$n = \frac{pV}{RT}$$

Pressure, p = 60.0 atm = $60.0\text{atm}\left(\frac{1.013\times10^5\text{Pa}}{1\text{atm}}\right) = 6.08\times10^6\text{Pa}$

Volume, V = 7.5L = 0.0075 m^3

Temperature, T = 35.5 + 273.15 = 308.7 K

$$\text{So, } n = \frac{\text{pV}}{\text{RT}} = \frac{(6.08\times10^6\text{Pa})(0.0075\text{m}^3)}{8.314\text{J/K}(308.7\text{K})} = 17.76 \text{ moles}$$

The molecular mass is then: 79.59g/7.76 = 4.5g/mol

Note: This molecular mass is an average. For example, a mixture of helium and neon could have this average molecular mass.

Problem 4.- Determine the mass of neon (atomic mass = 20.18) contained in a 5.25 liter tank at 4.80 atm and 26.85 °C.

Solution: The number of moles is given by:

$$n = \frac{pV}{RT} = \frac{4.8(1.013\times10^5 Pa)(0.0052m^3)}{(8.314J/K)(300K)} = 1.014 \text{ moles}$$

Since each mole has a mass of 20.18g, the amount of Ne is 1.014×20.18g = **20.46g**

Problem 5.- Glycine is an amino acid and does not behave like an ideal gas, but when in the gas phase and very diluted its behavior can be approximated by the pV=nRT equation. Calculate the density of glycine in the gas phase at T = 180ºC at a pressure of 1.5×10^{-5} torr.

1 atm = 760 torr

Solution: The density:

$$\frac{N}{V} = \frac{p}{k_B T} = \frac{(1.5\times10^{-5} torr)(1atm/760torr)(1.013\times10^5 Pa/1atm)}{(1.38\times10^{-23} J/K)(180+273)K} = \mathbf{3.20\times10^{17}/m^3}$$

This is in number of molecules per cubic meter.

Problem 6.- 240 J of heat is added to a monatomic ideal gas under conditions of constant volume, resulting in a temperature rise of 12K. How much heat will be required to produce the same temperature change, but at constant pressure?

Solution: The heat capacity at constant pressure is 5/3 times the heat capacity at constant volume for a monatomic gas, so the heat necessary will be 5/3×240 = **400J**