Thermal Physics

Two-level system

Problem 1.- Consider a system that can only exist in two states, one with energy zero and the other with energy $\varepsilon = 1eV$
a) Find an expression for the average energy.
b) Find an expression for the heat capacity by taking the derivative of the energy.
c) Calculate the heat capacity when $k_B T = 1eV$

Solution:

a) $\langle E \rangle = \dfrac{\varepsilon e^{-\varepsilon / k_B T}}{1 + e^{-\varepsilon / k_B T}} = \dfrac{\varepsilon}{e^{\varepsilon / k_B T} + 1}$

b) $C = \dfrac{\partial}{\partial T} \dfrac{\varepsilon}{e^{\varepsilon / k_B T} + 1} = k_B \dfrac{e^{\varepsilon / k_B T}}{\left(e^{\varepsilon / k_B T} + 1\right)^2} \left(\dfrac{\varepsilon}{k_B T} \right)^2$

c) $C = \dfrac{\partial}{\partial T} \dfrac{\varepsilon}{e^{\varepsilon / k_B T} + 1} = k_B \dfrac{e}{(e+1)^2} = 2.7 \times 10^{-24} J / K$