Thermal Physics

Nanowire

Problem 1.- Consider a 1-Dimensional ideal gas of electrons as a model for the conduction electrons in a nanowire.

a) Find the Fermi energy of electrons.
b) What is the Fermi velocity of the conduction electrons?

[Use a linear density corresponding to the lattice spacing of sodium of N/L=4.02×10^9/m and the mass of the electron $m_e = 9.1\times10^{-31}\,\text{kg}$]

Solution: An electron confined to a one-dimensional wire will have its momentum quantized as follows:
$p = \frac{h}{\lambda}$, but λ can only have values $\frac{2L}{n}$, where L is the length of the wire and n is a positive integer. The energy in one dimension is quantized also:

$$\varepsilon = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{(nh/2L)^2}{2m} = n^2\frac{h^2}{8mL^2}$$

Each energy level can contain two electrons due to spin multiplicity, so:
$n_F = \frac{N}{2}$

Then, the Fermi energy is: $\varepsilon_F = \frac{N^2h^2}{32mL^2}$

In case of a density N/L=4.02×10^9/m we get:

$$\varepsilon_F = \frac{(4.02\times10^9/\text{m})^2(6.62\times10^{-34}\,\text{Js})^2}{32(9.1\times10^{-31}\text{kg})} = 2.43\times10^{-19}\,\text{J} = \mathbf{1.52\ eV}$$

To get the Fermi velocity, notice that this Fermi energy is the kinetic energy, so:

$$\frac{1}{2}mv_F{}^2 = 2.43\times10^{-19}\,\text{J} \rightarrow v_F = \sqrt{\frac{2(2.43\times10^{-19}\,\text{J})}{m}} = \sqrt{\frac{2(2.43\times10^{-19}\,\text{J})}{(9.1\times10^{-31}\,\text{kg})}} = \mathbf{7.30\times10^5 m/s}$$

Problem 1a.- Consider a 1-Dimensional ideal gas of electrons as a model for the conduction electrons in a nanowire.
a) Find the Fermi energy,
b) Fermi velocity,
c) Fermi temperature, and
d) n_{Fermi}

Use a linear density corresponding to a lattice spacing of N/L=3.5×10^9/m, the mass of the electron $m_e = 9.1\times10^{-31}$kg and L=1 micron

Solution: The number of electrons is 3,500 given the lattice spacing and the length of the wire. This also considers that each atom contributes 1 electron to conduction as in a valence 1 metal.

With this number we can find the n_F: $n_F = 1750$, where the factor of 2 is due to spin.

Next the Fermi energy $E_F = n_F^{\ 2}\dfrac{h^2}{8mL^2} = 1.84\times10^{-19}J$

To find the velocity we use the kinetic energy equation

$$E_F = \frac{1}{2}mv_F^{\ 2} \rightarrow v_F = \sqrt{\frac{2E_F}{m}} = 6.37\times10^5 m/s$$

Alternatively using De Broglie equation

$$p = mv_F = \frac{h}{\lambda} = \frac{nh}{2L} \rightarrow v_F = \frac{nh}{2Lm} = 6.37\times10^5 m/s$$

And using Boltzmann constant we calculate the Fermi temperature

$$T_F = \frac{E_F}{k_B} = 13{,}400K$$