Thermal Physics

Paramagnetism

Problem 1.- Consider a system of N independent spins in a magnetic field B. Find the expected value of the fractional magnetization 2s/N, where s is the excess s.
You will need to change variables, s for U in the degeneracy equation and later back to s.
Use the answer of the problem to tell what the fractional magnetization will be for $\tau \to \infty$ and for $\tau \to 0$

Solution: We will use the approximation:

$$g(N,s) = \sqrt{\frac{2}{\pi N}}\, 2^N e^{-2s^2/N}$$

Then, the entropy is:

$$\sigma = \log(g(N,s)) = \log\left(\sqrt{\frac{2}{\pi N}}\, 2^N\right) - \frac{2s^2}{N}$$

We change variables from "s" to energy:

$U = -2msB \to s = -\frac{U}{2mB}$, getting:

$$\sigma = \log\left(\sqrt{\frac{2}{\pi N}}\, 2^N\right) - \frac{2\left(-\frac{U}{2mB}\right)^2}{N} = \log\left(\sqrt{\frac{2}{\pi N}}\, 2^N\right) - \frac{U^2}{2m^2B^2N}$$

According to the definition of temperature:

$$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N = \left(\frac{\partial\left(\log\left(\sqrt{\frac{2}{\pi N}}\, 2^N\right) - \frac{U^2}{2m^2B^2N}\right)}{\partial U}\right)_N = -\frac{U}{m^2B^2N}$$

Now, changing the variable back to spin ($U = -2msB$)

$$\frac{1}{\tau} = -\frac{-2msB}{m^2B^2N} = \frac{2s}{mBN} \to \quad \frac{2s}{N} = \frac{mB}{\tau}$$

This important result called “Curie’s Law”. It shows that the fractional magnetization is proportional to the external magnetic field and inversely proportional to the absolute temperature.

At very high temperatures, the equation tells us that the fractional magnetization will decay as 1/T, which corresponds very well with experimental results, however the equation tells us that the fractional magnetization will tend to infinity when the temperature goes to zero. This is not correct, because the maximum fraction that you can get in reality is 1.