Thermal Physics

Relativistic electrons in 2D

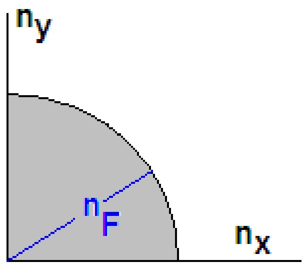
Problem 1.- For electrons whose dispersion relation is linear, the kinetic energy is given by:
$\varepsilon = pc$
Where p is the linear momentum and c is a constant speed.
Calculate the Fermi energy of N of these electrons, given the square area (L^2) where they are confined.

Solution: The momentum is quantized as in the non-relativistic case to values equal to:

$$p = \frac{h}{\lambda}, \text{ with } \lambda = \frac{2L}{n}, \text{ so } p = \frac{nh}{2L}$$

Where $n = \sqrt{n_x^2 + n_y^2}$

The number of electrons in the disk shown in the figure has to be equal to N:



So: $Area = \frac{\pi}{4} n_F^2 = number\ of\ levels$

Due to spin multiplicity each level can contain two electrons, so:

$$N = \frac{\pi}{2} n_F^2 \rightarrow n_F = \sqrt{\frac{2N}{\pi}}$$

The Fermi Energy for this system will be then: $\varepsilon_F = pc = \sqrt{\frac{N}{2\pi}} \frac{hc}{L}$

Problem 1a.- For electrons whose dispersion relation is linear, the kinetic energy is given by:
$\varepsilon = pc_g$
Where p is the linear momentum and c_g is a constant speed=10^6 m/s.
Calculate the Fermi energy of N=10,000 of these electrons, given that the square area (L^2=10^{-16} m^2) where they are confined.

Solution: The momentum is given by $p = \frac{h}{\lambda} = \frac{nh}{2L} \rightarrow E = \frac{nhc_g}{2L}$, and the value of n at the Fermi level is

$$10{,}000 = 2\times\frac{\pi}{4}n_F{}^2 \rightarrow n_F = \sqrt{\frac{2\times 10{,}000}{\pi}} = 80$$

Then the Fermi energy will be $E_F = \frac{nhc_g}{2L} = 2.64\times 10^{-18}\,J$