Thermal Physics

Heat Capacity of Blackbody Radiation

Problem 1.- For 1 m^3 of blackbody radiation at room temperature (300K) find

a) The heat capacity C_V,
b) The number of atoms of a monatomic gas that would give the same C_V.

Solution:
a) The energy stored in the electromagnetic waves of a cavity at temperature T is given by:

$$U = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} V T^4$$

Therefore, the heat capacity will be:

$$C_V = \frac{\partial U}{\partial T} = \frac{4\pi^2 k_B^4}{15\hbar^3 c^3} V T^3$$

Given the volume of $1m^3$ and T=300K we get:

$$C_V = \frac{4\pi^2 k_B^4}{15\hbar^3 c^3} V T^3 = \frac{4(3.1416)^2 (1.38\times10^{-23} J/K)^4}{15(1.055\times10^{-34} Js)^3 (3\times10^8 m/s)c^3}(1m^3)(300K)^3$$

$$C_V = 8.13\times10^{-8}\, J/K$$

Recall that the heat capacity of an ideal gas is $\frac{3}{2}Nk_B$, where N is the number of atoms,

so: $\frac{3}{2}Nk_B = 8.13\times10^{-8} \rightarrow N = 3.93\times10^{15}$
At standard temperature and pressure, this gas would occupy a volume of 0.14 mm^3.

Problem 2.- The energy from electromagnetic waves in equilibrium in a box is used to melt ice. If the absolute temperature of the box is increased by a factor of three, the mass of ice that can be melted in a fixed amount of time is increased by a factor of

(A) 3
(B) 8
(C) 27
(D) 81
(E) 243

Solution: Since the energy is proportional to the temperature to the 4^{th} power we can melt 3^4=**81** times the mass.

Problem 3.- Consider a volume of $1.5m^3$ of blackbody radiation at a temperature of 5800K find the amount of energy in the volume.

Solution: recall the Stefan-Boltzmann constant

$$\sigma T^4 = \frac{uc}{4} \rightarrow u = \frac{4\sigma T^4}{c}$$, where u is the energy density.

Then the energy in 1.5m^3 is

$$Energy = \frac{4\sigma T^4}{c} Volume = \frac{4\times 5.67\times 10^{-8}\times 5800^4}{3\times 10^8}\times 1.5 = \mathbf{0.86\ J}$$

Problem 4.- A star has a surface area A and absolute temperature $T = 6{,}000K$. You find another star with the same surface area, but it emits 16 times the power of the first one. What is the surface temperature of the second star?

(A) $3{,}000K$
(B) $6{,}000K$
(C) $12{,}000K$
(D) $24{,}000K$
(E) $96{,}000K$

Solution: (C)