Thermal Physics

Radiation Pressure

Problem 1.- The radiation field and a monatomic gas exist in thermal equilibrium in a region of space. If the density of the gas is the STP density of 2.69×10^{19} atoms/cm^3 (Loschmidt's number), what must be the temperature so that the gas pressure and the radiation pressure are equal?
In the interior of very high mass stars, the radiation pressure dominates the kinetic pressure.

Solution: The radiation pressure is:

$$p=\frac{\pi^2}{45\hbar^3c^3}\tau^4$$

For an ideal gas:

$pV = N\tau \rightarrow p=\frac{N}{V}\tau$ If the two pressures are equal then: $\frac{\pi^2}{45\hbar^3c^3}\tau^4=\frac{N}{V}\tau$

Solving for τ we get:

$$\frac{\pi^2}{45\hbar^3c^3}\tau^3=\frac{N}{V}\rightarrow\tau^3=\frac{N}{V}\frac{45\hbar^3c^3}{\pi^2}\rightarrow\tau=\sqrt[3]{\frac{N}{V}\frac{45\hbar^3c^3}{\pi^2}}$$

With the density given in the problem N/V=2.69×10^{19} atoms/cm^3, or N/V=2.69×10^{25} atoms/m^3 we get:

$$\tau=\sqrt[3]{2.69\times10^{25}\left(\frac{\text{atoms}}{\text{m}^3}\right)\frac{45(1.055\times10^{-34}\,\text{Js})^3(3\times10^8\,\text{m/s})^3}{(3.1416)^2}}=1.57\times10^{-17}\,\text{J}$$

To get the temperature in kelvin we need to divide τ by Boltzmann constant:

$T=\frac{\tau}{k_B}=$**1.14×10^6 K**.

This temperature is present in the interior of stars, where the radiation pressure dominates the kinetic pressure.

Problem 1a.- The radiation field and a very dilute monatomic gas exist in thermal equilibrium in a region of space at T=300K. Find the density of the gas if the pressures due to the atoms and the photons are equal.

Solution: If the pressures due to the atoms and the photons are equal then:

$$\frac{\pi^2}{45\hbar^3c^3}\tau^4=\frac{N}{V}\tau\rightarrow\frac{N}{V}=\frac{\pi^2}{45\hbar^3c^3}\tau^3=\frac{(3.1416)^2(1.38\times10^{-23}J/K\times300K)^3}{45(1.05\times10^{-34}\,\text{Js})^3(3\times10^8\,\text{m/s})^3}=\mathbf{4.98\times10^{14}/m^3}$$