Thermal Physics

Fermi Gas in 2D

Problem 1.- Calculate the density of states of electrons D(ε) living in a square of area A.

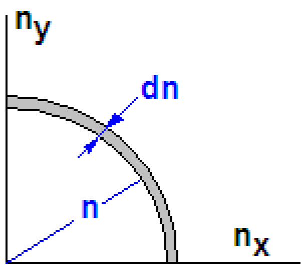
Solution: Analogous to the case of 3-D, the energy is:

$$\varepsilon = n^2 \frac{h^2}{8mL^2}, \qquad \text{Equation (1)}$$

Where $n = \sqrt{n_x^2 + n_y^2}$

In n-space, the allowed states are integer pairs (n_x, n_y) each distinct one able to be occupied by two electrons.

Observe the area in the following figure:



The shaded area is: $\text{Area} = \left(\frac{\pi}{2} n\right) dn$

The number of states in that area is twice that amount (2 per level), so:

$$\text{Number of states} = \pi n dn$$

Recall that the number of states is equal to $D(\varepsilon)d\varepsilon$ so:

$$D(\varepsilon)d\varepsilon = \pi n dn \qquad \text{Equation (2)}$$

Now we take a differential of the energy (from Equation (1) above):

$$d\varepsilon = \frac{h^2}{8mL^2} 2ndn \rightarrow dn = \frac{4mL^2 d\varepsilon}{nh^2}$$

And replacing this in Equation (2)

$$D(\varepsilon)d\varepsilon = \pi n \frac{4mL^2 d\varepsilon}{nh^2} \rightarrow D(\varepsilon) = 4\pi \frac{mL^2}{h^2}$$

Problem 2.- Calculate the density of states of electrons D(ε) at the Fermi level for a monatomic layer of sodium in the shape of a square of side 15nm and inter-atomic separation of 0.15nm

Solution: The number of electrons in the surface is 10,000 since there will be 100 atoms per side (assuming a simple square lattice). Then the Fermi number n_F is

$$10{,}000 = 2\times\frac{\pi}{4}n_F^{\ 2} \rightarrow n_F = \sqrt{\frac{2\times 10{,}000}{\pi}} = 80$$

We can also calculate the Fermi energy since

$$E = n^2\frac{h^2}{8mL^2} \rightarrow E_F = n_F^{\ 2}\frac{h^2}{8mL^2} = 1.7\times 10^{-18}\,J$$

For the density of states, consider a differential of n:

$$D(\varepsilon)dE = 2\times\frac{\pi}{2}ndn \rightarrow D(\varepsilon) = \pi n\frac{dn}{dE} = \pi n\frac{8mL^2}{2nh^2} = \frac{\pi n_F^{\ 2}}{2}\frac{1}{E_F} = 5.9\times 10^{21}\,J^{-1}$$

Problem 3.- Calculate the density of states of electrons D(ε) living in a square of area A at the Fermi level.

Use a linear surface corresponding to a lattice spacing of $N/L^2=12.25\times10^{18}/m^2$, the mass of the electron 9.1×10^{-31}kg and L=1 micron.

Solution: The number of electrons in the surface is 12.25×10^{6}, given the density and size. Then the Fermi number n_F is

$$12.25\times 10^6 = 2\times\frac{\pi}{4}n_F^{\ 2} \rightarrow n_F = \sqrt{\frac{2\times 12.25\times 10^6}{\pi}} = 2{,}793$$

We can also calculate the Fermi energy since

$$E = n^2\frac{h^2}{8mL^2} \rightarrow E_F = n_F^{\ 2}\frac{h^2}{8mL^2} = 4.69\times 10^{-19}\,J$$

For the density of states, consider a differential of n:

$$D(\varepsilon)dE = 2\times\frac{\pi}{2}ndn \rightarrow D(\varepsilon) = \pi n\frac{dn}{dE} = \pi n\frac{8mL^2}{2nh^2} = \frac{\pi n_F^{\ 2}}{2}\frac{1}{E_F} = 2.61\times 10^{25}\,J^{-1}$$