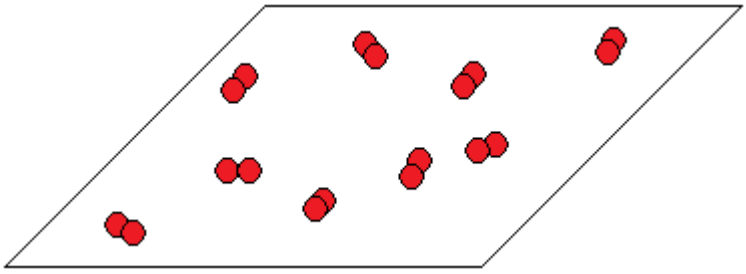
Thermal Physics

H_2 in 2D

Problem 1.- N molecules of hydrogen H_2 adsorbed on a flat surface of area A are in thermal equilibrium at temperature T. On the surface, they behave as a non-interacting two-dimensional gas. In particular, the rotational motion is confined to the plane of the surface. The quantum state of the planar rotation is specified by a single quantum number m, which can take on the values 0, ±1, ±2, ±3, ±4, etc.



There is one quantum state for each allowed value of m. The energies of the rotational states are $\varepsilon_m = \frac{\hbar^2}{2I} m^2$ where I is the moment of inertia.

a) Find an expression for the rotational partition function of a single molecule.

b) Find the ratio of the two probabilities p(m = 3)/p(m = 2)

Solution:

a) The partition function is the sum of all the Boltzmann factors:

$$Z = \sum_{m=-\infty}^{\infty} e^{-\frac{\hbar^2}{2I\tau} m^2}$$

b) The ratio of the two probabilities is equal to the ratio of the Boltzmann factors:

$$p(m = 3)/p(m = 2) = \frac{e^{-\frac{\hbar^2}{2I\tau}(3)^2}}{e^{-\frac{\hbar^2}{2I\tau}(2)^2}} = e^{-\frac{5\hbar^2}{2I\tau}}$$