Thermal Physics

Heat capacity of an ideal gas

1) We learn that a mono-atomic ideal gas has a heat capacity $\frac{3}{2}N$ in fundamental units, or $\frac{3}{2}Nk_B$ in conventional units. Normally this is written as:

$$C_V = \frac{3}{2}Nk_B = \frac{3}{2}\left(\frac{N}{N_A}\right)(N_A k_B) = \frac{3}{2}nR$$

Where N_A is Avogadro's number, n is the number of moles and R is the ideal gas constant (8.314 J mol^{-1} K^{-1}).

This means that the heat capacity of a mono-atomic ideal gas is **Cv = 3/2R** per mole.

2) **What happens if the gas is diatomic?** Gases like N_2 and O_2 are composed of diatomic molecules that can store energy in their rotational motion. At high temperatures, the contribution to the heat capacity is equal to N (or Nk_B in conventional units). So an ideal, diatomic gas will have a heat capacity equal to $\frac{5}{2}Nk_B$. Since air is a mixture of mainly diatomic gases, this is a good approximation for the atmosphere.

Diatomic gases can also store energy in their vibrational motion, but for that degree of freedom to contribute significantly, the temperature has to be high.

So an ideal diatomic gas has a heat capacity of **Cv = 5/2R** per mole

3) The heat capacities defined above are at constant volume, in which case all the heat goes to change the internal energy U. This is because of the first law of thermodynamics:

$$Q = W + \Delta U$$

At constant volume W=0, so $Q = \Delta U$.
You can use these heat capacities to calculate internal energy.

4) **What happens if you let the gas expand at constant pressure?** If you add heat to an ideal gas at constant pressure, part of the heat goes to increase the kinetic energy of the molecules or atoms and some produces work in the expansion. Because of this, the heat capacity of an ideal gas at constant pressure is higher than at constant volume. We use the symbol C_P for the former.

Consider n moles of an ideal gas at a constant pressure P_1, initial volume V_1 and initial temperature T_1. If you increase the temperature to $T_2=T_1+\Delta T$ the volume will increase to:

$V_2 = \frac{V_1 T_2}{T_1}$, so the work done will be:

$$W = P_1(V_2 - V_1) = P_1\left(\frac{V_1 T_2}{T_1} - V_1\right) = \frac{P_1 V_1}{T_1}(T_2 - T_1) = nR\Delta T$$

Again, using the first law of thermodynamics:

$$Q = W + \Delta U = nR\Delta T + \Delta U = nR\Delta T + C_V \Delta T$$

Dividing by ΔT we get: $\frac{Q}{\Delta T} = nR + C_V$

The left-hand side of the equation is by definition the heat capacity at constant pressure so:

$$C_P = nR + C_V$$

The heat capacity at constant pressure is nR higher than at constant volume.

5) **Summary:** The heat capacities per mole of ideal gases are:

$$\text{Monoatomic}\left\langle \begin{array}{l} C_V = \frac{3}{2}R \\ \\ C_P = \frac{5}{2}R \end{array}\right. \qquad \text{Diatomic}\left\langle \begin{array}{l} C_V = \frac{5}{2}R \\ \\ C_P = \frac{7}{2}R \end{array}\right.$$

6) **Definition of gamma:** The ratio of C_P to C_V is always greater than 1, and it is a very useful quantity. It is called gamma.

$$\gamma = \frac{C_P}{C_V}$$

Notice that for mono-atomic gases $\gamma = 5/3 = 1.67$ and for diatomic gases $\gamma = 7/5 = 1.4$

Air: A good approximation for dilute air at moderately high temperature is to consider $C_V = 5/2R$ per mole, $C_P = 7/2R$ per mole and a molecular weight of 29.0 u.