

Classical Mechanics

Vectors

Problem 1.- Consider 3 points in space A , B and C , and the three vectors from the origin to the points: a , b and c .

Determine the distance from the origin to the plane defined by the points A , B and C .

Solution: We can find a vector perpendicular to the plane by cross multiplication of two of its sides, for example:

$$\vec{V}_\perp = (a-b) \times (a-c) = a \times b + b \times c + c \times a$$

Then, if we calculate the dot-product of this vector with any of the other vectors we will have the desired distance multiplied by the magnitude of \vec{V}_\perp , so:

$$d = \frac{a \cdot \vec{V}_\perp}{|\vec{V}_\perp|} = \frac{a \cdot (a \times b + b \times c + c \times a)}{|a \times b + b \times c + c \times a|} = \frac{a \cdot b \times c}{|a \times b + b \times c + c \times a|}$$

Problem 2.- Consider 3 points in space A , B and C , and the three vectors from the origin to the points: a , b and c like in the previous problem.

Determine the area of the triangle ABC .

Solution: We can use the cross product equation to get the area:

$$A = \frac{|(a-b) \times (a-c)|}{2} = \frac{|a \times b + b \times c + c \times a|}{2}$$

Problem 3.- Evaluate the integral

$$I = \int A \times \ddot{A} dt$$

Solution: Consider the following derivative, where we apply the chain rule:

$$\frac{d}{dt} A \times \dot{A} = \dot{A} \times \dot{A} + A \times \ddot{A}$$

The first term in the right hand side of the equation is zero because it is a cross product of a vector with itself, so back in the integral we get:

$$I = \int A \times \ddot{A} dt = A \times \dot{A} + C$$

Problem 4.- For what values of a are the vectors $\vec{A} = 2a\hat{i} - 2\hat{j} + a\hat{k}$ and $\vec{B} = a\hat{i} + 2a\hat{j} + 2\hat{k}$ perpendicular?

Solution: The dot product has to be zero if the vectors are perpendicular, so:

$$\vec{A} \cdot \vec{B} = (2a\hat{i} - 2\hat{j} + a\hat{k}) \cdot (a\hat{i} + 2a\hat{j} + 2\hat{k}) = 0$$

$$\vec{A} \cdot \vec{B} = 2a^2 - 2a = 0,$$

The two solutions are $a = 0$ and $a = 1$.