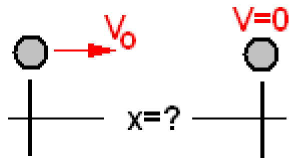
# Classical Mechanics

## Acceleration

**Problem 1.-** An object with initial velocity $v_o$ enters a region where the acceleration is $a = -kx^2$, calculate how far will it penetrate.

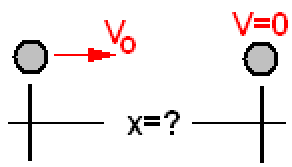



**Solution:** $a = -kx^2 \rightarrow \frac{dv}{dt} = -kx^2 \rightarrow v\frac{dv}{dx} = -kx^2 \rightarrow vdv = -kx^2dx$

And integrating we get:

$$\int_{v_o}^{0} vdv = -\int_{0}^{x} kx^2dx \rightarrow \frac{v_o^{\,2}}{2} = \frac{kx^3}{3} \rightarrow \quad x = \sqrt[3]{\frac{3v_o^2}{2k}}$$

**Problem 1a**.- An object with initial velocity $v_o$ enters a region where the acceleration is $a = -kx^5$, calculate how far will it penetrate.




**Solution:**

$$a = -kx^5 \rightarrow \frac{dv}{dt} = -kx^5 \rightarrow v\frac{dv}{dx} = -kx^5 \rightarrow vdv = -kx^5dx$$

And integrating we get:

$$\int_{v_o}^{0} vdv = -\int_{0}^{x} kx^5dx \rightarrow \frac{v_o^{\,2}}{2} = \frac{kx^6}{6} \rightarrow x = \sqrt[6]{\frac{3v_o^2}{k}}$$

**Problem 2.-** Careful measurements show that an object moving in very thick molasses has an acceleration given by:

$a = -bv^4$

Calculate the velocity as a function of time if the initial velocity is $v(0) = v_o$

**Solution:** By definition: $a = \frac{dv}{dt}$, so: $\frac{dv}{dt} = -bv^4$, separating variables: $-\frac{dv}{v^4} = bdt$

Integrating: $\frac{1}{3v^3} = bt + C$

Using the initial condition: $\frac{1}{3v_o^3} = C \rightarrow \frac{1}{3v^3} = bt + \frac{1}{3v_o^3}$,

Solving for v gives us: $v = \frac{v_o}{\sqrt[3]{1+3v_o^3 bt}}$

**Problem 3.-** A roller coaster uses magnetic brakes to slow down from an initial velocity of

$v_1 = 14.7 \mathrm{m/s}$ to $v_2 = 0.735 \mathrm{m/s}$.

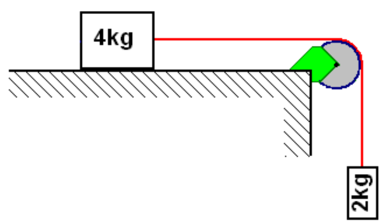
Calculate how long (time) it takes to do this, knowing that the acceleration can be written as:

$a = -bv$, where $b = 1.5$ Hz

**Solution:**

$$\frac{dv}{dt} = -bv \rightarrow \frac{dv}{v} = -bdt \rightarrow \ln\left(\frac{v_2}{v_1}\right) = -bt \rightarrow t = \frac{1}{b}\ln\left(\frac{v_1}{v_2}\right) = \frac{1}{1.5}\ln\left(\frac{14.7}{0.735}\right) = \mathbf{2\ seconds}$$

**Problem 4**.- Calculate the tension in string of the "half-Atwood machine" shown in the figure. Assume friction is negligible.




**Solution:** There is only one force accelerating the whole system: The weight of the 2kg mass, which is 19.6N and the acceleration is $a = \frac{F}{m} = \frac{19.6}{6} = 3.27 m/s^2$

We now can look at the 2kg mass. There are two forces acting, its weight and the tension in the string so Newton's second law says:

$$mg - F_{tension} = ma \rightarrow F_{tension} = m(g-a) = 2(9.8-3.27) = \mathbf{13.1\ N}$$

**Problem 5**.- A rocket has a mass of 100,000 kg out of which 95,000 kg is fuel. The speed of the gases generated by burning the fuel is 1,900 m/s. Calculate the final speed of the rocket after burning all its fuel starting from zero velocity. Only consider the thrust of the gases, no other forces.

**Solution:** The thrust is: $\mathrm{F}=-v_{gases}\dfrac{dm}{dt}$, and according to Newton's second law:

$\mathrm{F}=ma\rightarrow a=-\dfrac{v_{gases}}{m}\dfrac{dm}{dt}$, which can be written $\dfrac{\mathrm{dv}}{\mathrm{dt}}=-\dfrac{v_{gases}}{m}\dfrac{dm}{dt}\rightarrow \mathrm{dv}=-v_{gases}\dfrac{dm}{m}$

Integrating on both sides of the equation, we get:

$$\int_{0}^{\mathrm{v}}\mathrm{dv}=-v_{gases}\int_{100000}^{5,000}\frac{dm}{m}$$

That gives: $v=v_{gases}\left[\ln(100,000)-\ln(5,000)\right]=$ **5,700 m/s**