Classical Mechanics

Forced oscillator

Problem 1.- A flat horizontal plate driven from below oscillates vertically with amplitude of 1 mm. Some sand is sprinkled on the plate. The frequency of the oscillator is gradually increased from zero while keeping the amplitude constant. At what frequency will the sand lose contact with the plate? At what point in the cycle will this happen?

Solution: The vertical position can be described with the equation:

$x(t) = A\cos(\omega_o t - \delta)$, so the acceleration is $a = \frac{d^2x}{dt^2}x(t) = -A\omega_o^2\cos(\omega_o t - \delta)$,

Notice that the maximum acceleration is $A\omega_o^2$ and if this is equal to "g" the sand will lose contact with the ground.

So $A\omega_o^2 = g \rightarrow A(2\pi f)^2 = g \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{A}} = \frac{1}{2(3.1416)}\sqrt{\frac{9.8}{1\times10^{-3}}} =$ **15.8 Hz**

Since the maximum acceleration happens at the end of the motion, the sand will lose contact at the maximum height.

Problem 1a.- The motion of the ground during certain earthquake can be modeled as a sinusoidal vertical motion of frequency f=1.5Hz. Calculate the maximum amplitude of the earthquake that does not cause loose objects to lose contact with the ground.

Solution: The vertical position can be described with the equation:

$x(t) = A\cos(\omega_o t - \delta)$, so the acceleration is $a = \frac{d^2x}{dt^2}x(t) = -A\omega_o^2\cos(\omega_o t - \delta)$,

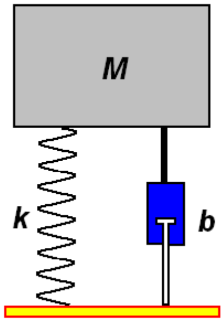
Notice that the maximum acceleration is $A\omega_o^2$ and if this is equal to "g" the objects will lose contact with the ground.

So $A\omega_o^2 = g \rightarrow A = \frac{g}{\omega_o^2} = \frac{g}{(2\pi f)^2} = \frac{9.8m/s^2}{(2\times3.1416\times1.5Hz)^2} =$ **0.11 m**

Problem 2.- Calculate the response of the damped oscillator shown in the figure after a long time of applying the force:

$F = 15\sin(5t)$

Indicate "x" as a function of time as your answer.



M=1.5kg, k=6 N/m and b=3 Ns/m

Solution: The response has the form:

$$x = A\sin(5t + \varphi)$$

and must satisfy the differential equation: $ma = -kx - bv + 15\sin(5t)$
we notice that

$$v = 5A\cos(5t + \phi) \text{ and}$$
$$a = -25A\sin(5t + \phi)\text{, so:}$$
$$ma = -kx - bv + 15\sin(5t)$$
$$\rightarrow -1.5\times 25A\sin(5t+\phi) = -6\times A\sin(5t+\phi) - 3\times 5A\cos(5t+\phi) + 15\sin(5t)$$

The amplitude is then:

$$A = \frac{15}{\sqrt{(1.5\times 25 - 6)^2 + (3\times 5)^2}} = \mathbf{0.43\ m}$$

And the angle:

$$\phi = \tan^{-1}\left(-\frac{3\times 5}{-1.5\times 25 + 6}\right) = 25.5^{\circ} + 180^{\circ} = \mathbf{205.5^{\circ}}$$