Classical Mechanics

Resonance

Problem 1.- Sketch the velocity as a function of the angular frequency for a damped oscillator with Q=10 and show that the full width of the curve between the points corresponding to $\frac{V_{max}}{\sqrt{2}}$ is approximately $\frac{\omega_o}{10}$.

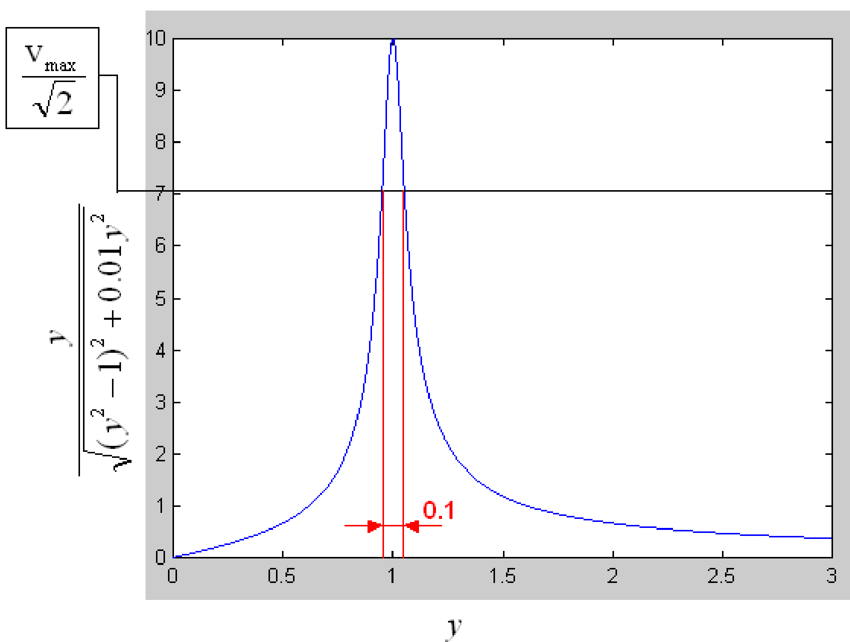
Solution: We plot the velocity amplitude: $\frac{A\omega}{\sqrt{(\omega^2-\omega_o^{\,2})^2+4\omega^2\beta^2}}$ for the case of Q=10, meaning that $\beta \approx \frac{\omega_o}{20}$, so the function is:

$$\frac{A\omega}{\sqrt{(\omega^2-\omega_o^{\,2})^2+4\omega^2(\omega_o/20)^2}}=\frac{A}{\omega_o}\frac{\omega/\omega_o}{\sqrt{(\omega^2/\omega_o^{\,2}-1)^2+0.01\omega^2/\omega_o^{\,2}}}$$

With the change of variable $y=\omega/\omega_o$, the amplitude becomes:

$$\frac{A}{\omega_o}\frac{y}{\sqrt{(y^2-1)^2+0.01y^2}}$$

So, yes the separation between the points that correspond to $\frac{V_{max}}{\sqrt{2}}$ is $\omega_o/Q=\omega_o/10$ which in terms of y corresponds to 0.1



Problem 2.- A small fiber vibrates like a simple harmonic oscillator with light damping. In one experiment you determine that the maximum amplitude occurs when the driving frequency is f=134 kHz, but the angular delay between driving force and oscillation reaches 90º at f=136 kHz. Based on these results, what is the natural frequency of oscillation in the absence of damping? and how much is β?

Solution: The angular delay between driving force and oscillation is:

$$\delta = \tan^{-1}\left(\frac{2\beta\omega}{\omega_o^2 - \omega^2}\right)$$

This value reaches 90º when $\omega_o = \omega$, so $\omega_o = 2\pi(136kHz) =$ **855 krad/s**

The resonance condition is $\omega = \omega_R = \sqrt{\omega_o^2 - 2\beta^2}$, so:

$$\beta = \sqrt{\frac{\omega_o^2 - \omega^2}{2}} = 2\pi\sqrt{\frac{136kHz^2 - 134kHz^2}{2}} =$$ **104 krad/s**