# Classical Mechanics

## Central force motion

**Problem 1**.- For a 2-body system, confirm numerically that the kinetic energy can be calculated by adding the individual kinetic energies or by adding the center of mass kinetic energy and the relative velocity energy.

For your numerical example, consider
$m_1 = 2$
$m_2 = 3$
$\vec{v}_1 = (20,25,5)$
$\vec{v}_2 = (-10,-5,0)$

**Solution:** First, the individual energies

$$KE = \frac{1}{2}m_1 v_1^{\,2} + \frac{1}{2}m_2 v_2^{\,2} = \frac{1}{2}\times 2\times(20^2+25^2+5^2)+\frac{1}{2}\times 3\times(10^2+5^2+0^2) = 1{,}237.5J$$

Now, the center of mass energy is $\frac{1}{2}M\dot{\vec{R}}^2$, where $M = m_1 + m_2 = 5$ is the total mass and $\dot{\vec{R}}^2 = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{2(20,25,5)+3(-10,-5,0)}{5} = (2,7,2)$, so the kinetic energy is $\frac{1}{2}\times 5\times(2^2+7^2+2^2) = 142.5J$

And the relative motion energy is $\frac{1}{2}\mu\dot{\vec{r}}^2$, where $\mu = \frac{m_1 m_2}{m_1+m_2} = 1.2$ is the reduced mass and $\dot{\vec{r}} = \vec{v}_1 - \vec{v}_2 = (20,25,5)-(-10,-5,0) = (30,30,5)$ is the relative velocity, so this kinetic energy is $\frac{1}{2}\times 1.2\times(30^2+30^2+5^2) = 1{,}095J$

The total is $1{,}095J + 142.5J = 1{,}237.5J$

We indeed observe that $\frac{1}{2}m_1 v_1^{\,2} + \frac{1}{2}m_2 v_2^{\,2} = \frac{1}{2}M\dot{\vec{R}}^2 + \frac{1}{2}\mu\dot{\vec{r}}^2$

**Problem 2.-** Use a computer program to plot the trajectory of a satellite in orbit around our planet given the following initial conditions:

$$initial - altitude = 500km$$
$$v_r = 0$$
$$v_\theta = r\dot{\theta} = 8{,}800 m/s$$

**Solution:**