Classical Mechanics

Gravitation

Problem 1.- Calculate the minimum speed that a particle must have on the surface of the sun to escape its gravitational attraction and leave the solar system. The mass of the sun is 1.99×10^{30} kg and its radius is 6.96×10^{8} m.

Solution:
If the particle is going to escape, it needs to have zero total energy at least:

$$K.E. = -P.E. \rightarrow \frac{1}{2}mv^2 = G\frac{Mm}{r} \rightarrow v = \sqrt{2GM/r}$$

With the values of the problem:

$$v = \sqrt{2(6.67\times10^{-11})(1.99\times10^{30})/(6.96\times10^{8})} = 618{,}000 \text{ m/s}$$

Problem 1a.- Calculate the minimum speed that a particle must have on the surface of Pluto to escape its gravitational attraction and leave. The mass of Pluto is 1.305×10^{22} kg and its radius is 1.15×10^{6} m.

Solution:
If the particle is going to escape, it needs to have zero total energy at least:

$$K.E. = -P.E. \rightarrow \frac{1}{2}mv^2 = G\frac{Mm}{r} \rightarrow v = \sqrt{2GM/r}$$

With the values of the problem:

$$v = \sqrt{2(6.67\times10^{-11})(1.305\times10^{22})/(1.15\times10^{6})} = 1{,}228 \text{ m/s}$$

Problem 2.- Calculate the kinetic energy of a satellite that has a mass of 355 kg in circular orbit around the Earth at a distance of 400 km above the surface.

Solution: To be in circular orbit the gravitational potential should be minus two times the kinetic energy, so:

$$K.E. = -\frac{P.E}{2} = G\frac{Mm}{2r} = 6.67\times10^{-11}\frac{(5.98\times10^{24})(355)}{2(6.38\times10^{6}+400\times10^{3})} = 1.04\times10^{10} \text{ J}$$

Problem 2a.- Calculate the kinetic energy of a satellite that has a mass of 84 kg in circular orbit around the Earth at a distance of 550 km above the surface.

Solution: In circular orbit the kinetic energy is minus ½ the gravitational potential:

$$K.E. = -\frac{P.E}{2} = G\frac{Mm}{2r} = 6.67\times10^{-11}\frac{(5.98\times10^{24})(84)}{2(6.38\times10^{6}+550\times10^{3})} = 2.42\times10^{9} \text{ J}$$

Problem 3.- Earlier we found that the minimum speed of a particle to escape the solar system from the surface of the Sun was 618,000 m/s. Calculate the speed of such a particle when it reaches the orbit of the Earth, which is 1.5×10^{11}m from the sun. The mass of the sun is 1.99×10^{30} kg and its radius is 6.96×10^{8} m.

Solution: The total energy of the particle is zero, so when it reaches our orbit we have:

$$-G\frac{m_{particle}m_{sun}}{r_{orbit}}+\frac{1}{2}m_{particle}v^2=0\rightarrow G\frac{m_{sun}}{r_{orbit}}=\frac{1}{2}v^2$$

$$\rightarrow v=\sqrt{\frac{2Gm_{sun}}{r_{orbit}}}=\sqrt{\frac{2(6.67\times10^{-11})(1.99\times10^{30})}{1.5\times10^{11}}}=\textbf{42.1 km/s}$$

Problem 4.- Calculate the minimum distance between the surface of the Earth and a satellite that has a period of 24 hours and an eccentricity of 0.25

Give your answer in terms of the radius of the earth R_E

Consider the period of a hypothetical satellite of radius R_E to be 80 minutes.

Solution: We first use Kepler's third law to find "a", the semi-major axis of the elliptical orbit:

$$\frac{T_1^{\ 2}}{a^3}=\frac{T_2^{\ 2}}{R_E^{\ 3}}\rightarrow a=R_E\sqrt[3]{\frac{T_1^{\ 2}}{T_2^{\ 2}}}=R_E\sqrt[3]{\frac{(24\times60)^2}{80^2}}=6.87R_E$$

The maximum approach to the center of the Earth will be:

$$r_{min}=a(1-\varepsilon)=6.87R_E(1-0.25)=5.15R_E$$

Finally, the minimum distance to the surface of the Earth will be **$4.15R_E$**