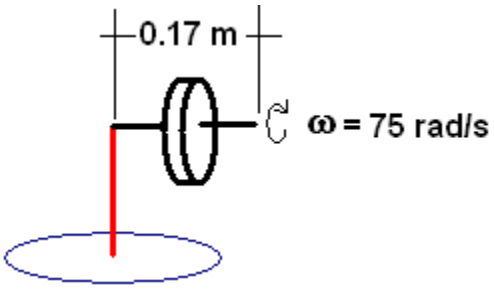
Classical Mechanics

Precession

Problem 1.- Consider a gyroscope consisting of a disk of mass 0.25kg and radius 0.055m mounted at the center of an axle 0.17m long. The gyroscope spins at 75 radians/s. Calculate:

a) The angular momentum of the disk
b) The torque produced by the weight of the disk
c) How long it takes for the gyroscope to precess once around.



Solution:

The moment of inertia is: $I = \frac{MR^2}{2}$ and then the angular momentum will be:

$$L = I\omega = \frac{MR^2}{2}\omega = \frac{(0.25kg)(0.055m)^2}{2}75rad/s = \mathbf{0.0284\ kgm^2s^{-1}}$$

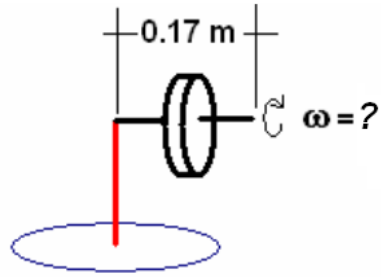
The torque produced by the weight of the disk:
$\tau = Fr\sin\theta = mgr = 0.25kg \times 9.8m/s^2 \times (0.17m/2) = \mathbf{0.208\ Nm.}$

How long it takes for the gyroscope to precess once around:

$$T = \frac{2\pi L}{\tau} = \frac{2(3.1416)(0.0284kgm^2/s)}{0.208Nm} = \mathbf{0.86\ s}$$

Problem 2.- Consider a gyroscope consisting of a disk of mass 0.25kg and radius 0.055m mounted at the center of an axle 0.17m long. Calculate how fast it needs to spin if it is going to take 3.5s to precess once around.

Solution:



The gyroscope has an angular momentum of $L = I\omega = \frac{1}{2}MR^2\omega$ and the torque due to its weight is: $\tau = \frac{dL}{dt} = Mgr$, where r is the distance from the center of mass to the pivot ($r = \frac{0.17m}{2} = 0.085m$ in this problem) to complete one turn we need to cover $2\pi L$, so:

$$\frac{dL}{dt}T = 2\pi L \rightarrow MgrT = 2\pi\frac{1}{2}MR^2\omega \rightarrow \omega = \frac{grT}{\pi R^2} = \frac{(9.8m/s^2)(0.085m)(3.5s)}{3.1416(0.055m)^2} = \mathbf{307\ rad/s}$$