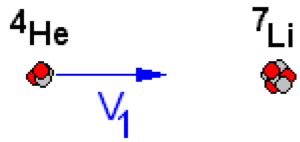
Classical Mechanics

Collisions

Problem 1.- An alpha particle moving to the right at v_1=23,100 m/s collides elastically and head on with a Lithium-7 nucleus that was initially at rest and bounces back. Calculate the velocity of the two particles after the collision.



Solution:

First equation, conservation of momentum: $4\times 23{,}100 = 4v_1' + 7v_2'$

Second equation, conservation of kinetic energy: $23{,}100 + v_1' = v_2'$

By substitution: $4\times 23{,}100 = 4v_1' + 7(23{,}100 + v_1') \rightarrow v_1' = \frac{4-7}{4+7}\times 23{,}100 =$ **-6,300 m/s**

And in the second equation: $v_2' = v_1' + 23{,}100 =$ **16,800 m/s**

Problem 2.- An alpha particle collides elastically and head on with an aluminum nucleus and bounces back. Calculate the velocity of the two particles after the collision.

Solution:
Conservation of linear momentum implies:
$m_{Al}v_{Al} + m_\alpha v_\alpha = m_{Al}v'_{Al} + m_\alpha v'_\alpha$

Given that the mass of aluminum is 27 amu and the mass of an alpha particle is 4 amu, we get:
$27v_{Al} + 4v_\alpha = 27v'_{Al} + 4v'_\alpha$ …. Equation 1

Conservation of kinetic energy implies:
$(v_{Al} - v_\alpha) = -(v'_{Al} - v'_\alpha)$
$v_{Al} - v_\alpha = -v'_{Al} + v'_\alpha$ …. Equation 2

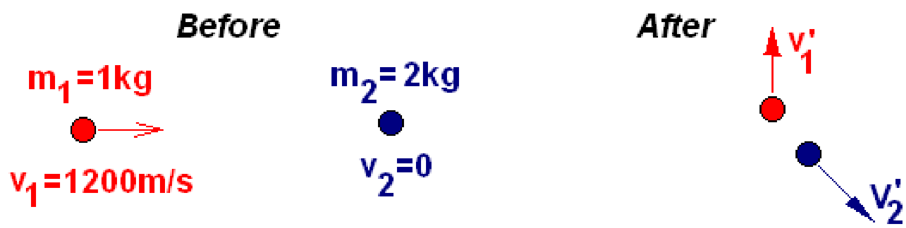
Solving for v'_{Al} we get:

$$v'_{Al} = \frac{\begin{vmatrix} 27v_{Al} + 4v_\alpha & 4 \\ v_{Al} - v_\alpha & 1 \end{vmatrix}}{\begin{vmatrix} 27 & 4 \\ -1 & 1 \end{vmatrix}} = \frac{27v_{Al} + 4v_\alpha - 4(v_{Al} - v_\alpha)}{27+4} = \frac{23}{31}v_{Al} + \frac{8}{31}v_\alpha$$

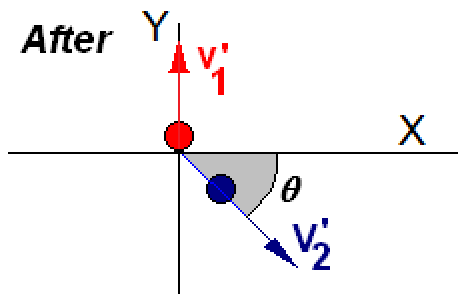
Solving for v'_α we get:

$$v'_\alpha = \frac{\begin{vmatrix} 27 & 27v_{Al} + 4v_\alpha \\ -1 & v_{Al} - v_\alpha \end{vmatrix}}{\begin{vmatrix} 27 & 4 \\ -1 & 1 \end{vmatrix}} = \frac{27(v_{Al} - v_\alpha) + 27v_{Al} + 4v_\alpha}{27+4} = \frac{54}{31}v_{Al} - \frac{23}{31}v_\alpha$$

Problem 3.- Calculate the velocity of the particles after the totally elastic collision shown in the figure:



Solution: A schematic after the collision



Conservation of momentum applies to both directions X and Y, so:

$$X: \quad m_1 v_1 = m_2 v'_2 \cos\theta$$
$$Y: \quad 0 = m_1 v'_1 - m_2 v_2 \sin\theta$$

With the values of the problem we can simplify these equations to give:

$$v_1 = 2v'_2 \cos\theta \rightarrow v'_2 = \frac{v_1}{2\cos\theta} = \frac{600}{\cos\theta}$$

$$0 = v'_1 - 2v'_2 \sin\theta \rightarrow v'_1 = 2v'_2 \sin\theta = 1200\tan\theta$$

Conservation of kinetic energy implies: $\frac{1}{2}m_1 v_1{}^2 = \frac{1}{2}m_1 v'_1{}^2 + \frac{1}{2}m_2 v'_2{}^2$

And replacing the results from above, we get:

$$v_1{}^2 = v'_1{}^2 + 2v'_2{}^2 \rightarrow 1200^2 = 1200^2 \tan^2\theta + 2\times\frac{600^2}{\cos^2\theta}$$

$$\rightarrow 4 = \frac{4\sin^2\theta + 2}{\cos^2\theta} = \frac{6 - 4\cos^2\theta}{\cos^2\theta}$$

$$\rightarrow 4\cos^2\theta = 6 - 4\cos^2\theta$$

$$\rightarrow 8\cos^2\theta = 6 \rightarrow \theta = \cos^{-1}\left(\sqrt{\frac{6}{8}}\right) = \mathbf{30^o}$$

Knowing the angle, we can calculate the velocities:

$$v'_1 = 1200\tan 30^\circ = \mathbf{693\ m/s}$$

$$v'_2 = \frac{600}{\cos 30^\circ} = \mathbf{693\ m/s}$$