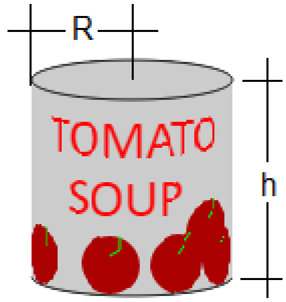
Classical Mechanics

Imposed Conditions

Problem 1.- Find the size of a can (R and h) that contains the maximum possible volume, given that its surface area is $0.0300m^2$.



Solution: This problem can be solved using Lagrange multipliers. The volume of the can is:

$$Volume = \pi R^2 h$$

And the area gives us the imposed condition:

$$Area = 2\pi Rh + 2\pi R^2 = 0.0300m^2 \rightarrow \phi = 2\pi Rh + 2\pi R^2 - 0.0300m^2 = 0$$

According to the method, the maximum value will happen when:

$$\frac{\partial Volume}{\partial R} = \lambda \frac{\partial \phi}{\partial R} \quad \text{and} \quad \frac{\partial Volume}{\partial h} = \lambda \frac{\partial \phi}{\partial h}$$

Then the equations are:

$$2\pi Rh = \lambda(2\pi h + 4\pi R) \quad \text{and} \quad \pi R^2 = \lambda(2\pi R)$$

Dividing one equation by the other (to eliminate lambda):

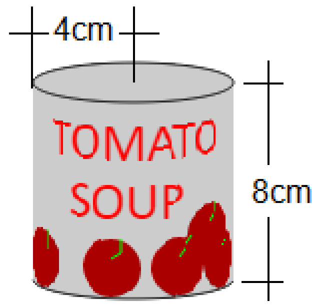
$$\frac{2\pi Rh}{\pi R^2} = \frac{\lambda(2\pi h + 4\pi R)}{\lambda(2\pi R)} \rightarrow 2\frac{h}{R} = \frac{h+2R}{R} \rightarrow 2h = h + 2R \rightarrow h = 2R$$

Therefore, to get the maximum volume the height of the can has to be equal to the diameter (how often do you see that in the supermarket?)

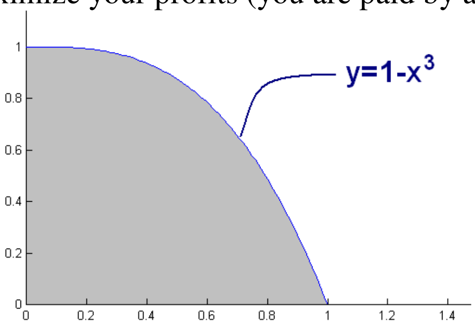
In our problem, this implies that:

$$Area = 2\pi Rh + 2\pi R^2 = 0.0300m^2 \rightarrow 6\pi R^2 = 0.0300m^2 \rightarrow R = \sqrt{\frac{0.0300m^2}{6\pi}} = \mathbf{0.04m}$$

And h=**0.08m**



Problem 2.- You are in the scrap metal business. You buy metal sheets cut in the shape shown in the figure for pennies and sell rectangular shapes for top dollar. Calculate the size of the rectangular shape that will maximize your profits (you are paid by area).



Solution: The area that you want to maximize is given by:

$F = xy$

And the imposed condition is $y = 1 - x^3 \rightarrow \phi = y + x^3 - 1 = 0$

Using the method of Lagrange multipliers, we need to have:

$\frac{\partial F}{\partial x} = \lambda \frac{\partial \phi}{\partial x}$ and $\frac{\partial F}{\partial y} = \lambda \frac{\partial \phi}{\partial y}$ for a maximum area.

Writing down the equations:

$y = \lambda(3x^2)$ and $x = \lambda 1$

Replacing the second equation in the first we get:

$y = (x)(3x^2) \rightarrow y = 3x^3$

And combining this equation with the imposed condition:

$y + x^3 - 1 = 0 \rightarrow 3x^3 + x^3 - 1 = 0 \rightarrow 4x^3 = 1 \rightarrow x = \sqrt[3]{0.25} =$ **0.63m**

and the value of y is:

$$y = 3x^3 = 3(\sqrt[3]{0.25})^3 = \mathbf{0.75m}$$