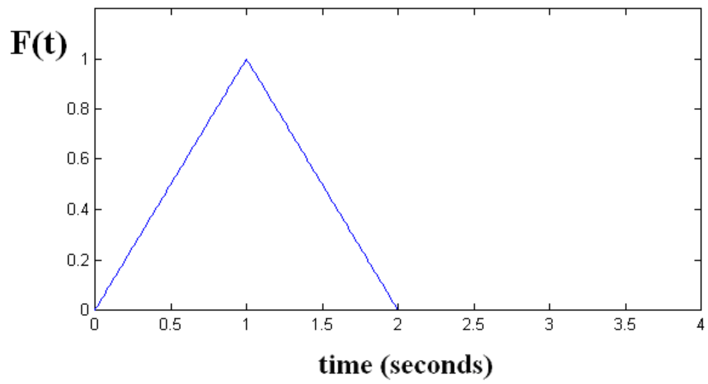
Classical Mechanics

La Place transform

Problem 1.- Find the Laplace transform of the pulse shown in the figure:



Solution: By definition:

$$f(p) = \int_0^{\infty} e^{-pt} F(t)dt$$

And the function is given by: $F(t) = \begin{cases} t & if \quad 0 < t < 1 \\ 2-t & if \quad 1 < t < 2 \\ 0 & otherwise \end{cases}$

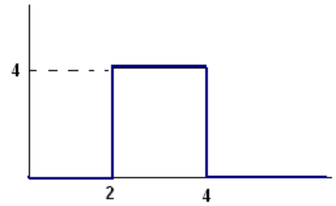
So the Laplace transform becomes:

$f(p) = \int_0^1 e^{-pt} t dt + \int_1^2 e^{-pt}(2-t)dt$, which can be integrated by parts:

$$f(p) = \int_0^1 \frac{tde^{-pt}}{-p} + \int_1^2 \frac{(2-t)de^{-pt}}{-p} = \left.\frac{te^{-pt}}{-p}\right|_0^1 + \int_0^1 \frac{e^{-pt}dt}{p} + \left.\frac{(2-t)e^{-pt}}{-p}\right|_1^2 + \int_1^2 \frac{e^{-pt}d(2-t)}{p}$$

$$f(p) = -\frac{e^{-p}}{p} + \left.\frac{e^{-pt}}{-p^2}\right|_0^1 + \frac{e^{-p}}{p} - \left.\frac{e^{-pt}}{-p^2}\right|_1^2 = \frac{e^{-p}}{-p^2} + \frac{1}{p^2} + \frac{e^{-2p}}{p^2} - \frac{e^{-p}}{p^2} = \frac{\left(1-e^{-p}\right)^2}{p^2}$$

Problem 2.- Find the Laplace transform of the function shown in the figure:



Solution: The easiest way of finding the transform is to notice that the function can be written as:

$f(t) = 4B(t-2) - 4B(t-4)$

Where B(t) is the "step function" $B(t) = \begin{cases} 0 & if \quad t < 0 \\ 1 & otherwise \end{cases}$

So: $f(s) = \dfrac{4e^{-2s}}{s} - \dfrac{4e^{-4s}}{s}$