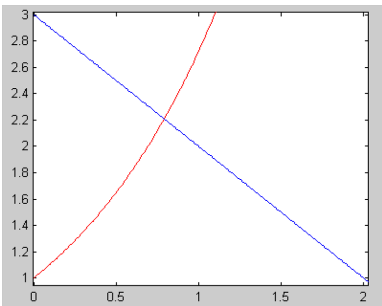
Classical Mechanics

Successive Approximations

Problem 1.-: Obtain the solution with 4 significant figures of the equations:

1) $e^x = 3 - x$
2) $e^{-x^2} = x + 0.5$

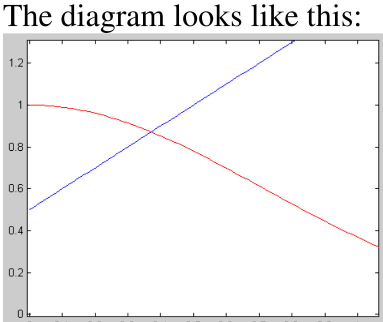
Solution: 1) $e^x = 3 - x$, the diagram looks like this:



An initial approximation could be x=1, which put on the right hand side of the equation gives us:
$e^x = 3 - 1 = 2 \rightarrow x_1 = \ln(2) = 0.6931$
Using this to get the next approximation:
$e^x = 3 - 0.6931 = 2.3068 \rightarrow x_2 = \ln(2.3069) = 0.8359$
And repeating the process we get:
$x_n =$ 1.0000, 0.6931, 0.8359, 0.7720, 0.8011, 0.7880, 0.7939, 0.7912, 0.7924, 0.7919, 0.7921, 0.7920, 0.7921, 0.7921

So, with four significant figures: x=0.7921

2) $e^{-x^2} = x + 0.5$
The diagram looks like this:



An initial approximation could be x=0.4, which put on the *left* hand side of the equation gives us:
$e^{-0.4^2} = x + 0.5 \rightarrow$
$x_1 = e^{-0.4^2} - 0.5 = 0.3521$
Using this to get the next approximation:

$e^{-0.3521^2} = x + 0.5 \rightarrow$

$x_1 = e^{-0.3521^2} - 0.5 = 0.3834$

Repeating the process we get:

$x_n =$ 0.4000, 0.3521, 0.3834, 0.3633, 0.3763, 0.3679, 0.3734, 0.3699, 0.3721, 0.3707, 0.3716, 0.3710, 0.3714, 0.3711, 0.3713, 0.3712

So, with four significant figures: x=0.3712

Note: starting with x=0.4 and replacing this on the right hand side of the equation will not converge.

Problem 2.- Obtain the solution with 4 significant figures of the equation:

$$\tan(x) = 3 - x \quad \text{for} \quad 0 < x < \pi/2$$

Solution: Assuming we know an approximate solution x_n we can estimate the next approximation with:

$x_{n+1} = \tan^{-1}(3 - x_n)$

Starting with x=1 we get:

$x_2 = \tan^{-1}(3-1) = 1.107$

$x_3 = \tan^{-1}(3-1.107) = 1.085$

$x_4 = \tan^{-1}(3-1.085) = 1.089$

$x_5 = \tan^{-1}(3-1.089) = 1.088$

$x_6 = \tan^{-1}(3-1.088) = 1.087$

$x_7 = \tan^{-1}(3-1.087) = 1.087$

And the solution converges to **x=1.087**

Problem 3.- Use successive approximations to obtain the solution with 4 significant figures of the equations:

a) $e^x = 10 - x$

b) $\sin x = 1 - x^3$