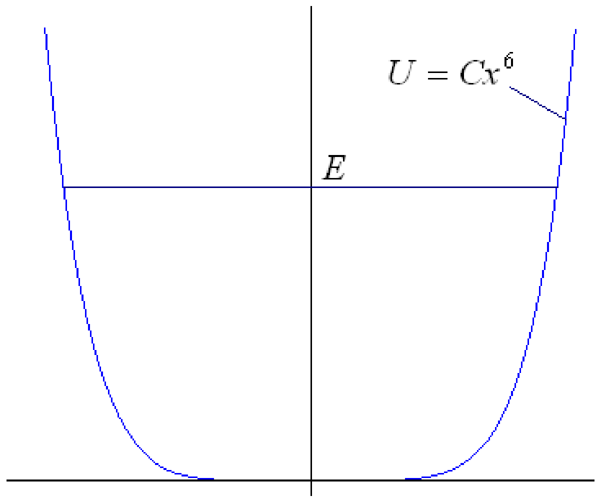
Classical Mechanics

Virial Theorem

Problem 1.- Use the virial theorem to calculate the average over time of the kinetic energy of a particle trapped in a one dimensional potential described by the equation:

$U = Cx^6$ if the total energy of the particle is E.

Is the result what you expected? Why or why not?



Solution: According to the virial theorem for a central potential whose equation is $U = Cx^n$, the average kinetic energy is given by: $\langle K.E. \rangle = \frac{n}{2} \langle P.E. \rangle$, and since n=6 in this problem we have:

$\langle K.E. \rangle = 3 \langle P.E. \rangle$, but we also know that $\langle K.E. \rangle + \langle P.E. \rangle = E$, so $\langle K.E. \rangle = \frac{3E}{4}$

Is this reasonable? Take into account that the potential is almost zero for most of the trajectory of the particle when moving back and forth between the turning points, so you expect the average kinetic energy to be larger than the average potential energy. Then yes, this is reasonable.

Also, notice that in the limit of $n \to \infty$ the walls would be vertical at x=1 and x=-1, and at that point all the average energy would be kinetic, like in the case of the particle in a one dimensional box that you study in quantum mechanics.

Problem 2.- Careful experimental observations of a particle trapped in a central potential of the form $P.E. = kr^n$ show that its average kinetic energy is 4/5 of the total energy. Calculate the exponent n in the potential energy expression.

Solution:

The virial theorem applied to a potential of the form $P.E. = kr^n$ establishes that:

$\langle K.E. \rangle = \frac{n}{2} \langle P.E \rangle$ …. Equation 1

However, we also know that the total energy is the sum of potential and kinetic energy, so:

$\langle K.E. \rangle + \langle P.E \rangle = E$

According to the problem $\langle K.E. \rangle = \frac{4}{5}E$, so $\langle P.E. \rangle = \frac{1}{5}E$ and replacing these values in equation 1 we get:

$\frac{4E}{5} = \frac{n}{2}\frac{E}{5} \rightarrow$ ***n=8***