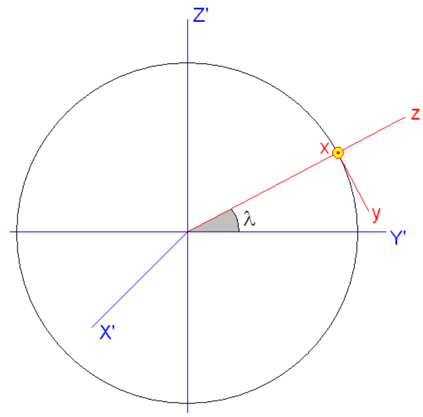
Classical Mechanics

g vector



In the geometry represented above

$$\vec{g} = -g_{\circ}\hat{z} + \omega^2 R_E \cos\lambda[\sin\lambda\hat{y} + \cos\lambda\hat{z}]$$

$$\alpha = \cos^{-1}\left(-\frac{\vec{g}\cdot\hat{z}}{|\vec{g}|}\right) = \cos^{-1}\left(\frac{g_{\circ} - \omega^2 R_E \cos^2\lambda}{\sqrt{(g_o - \omega^2 R_E \cos^2\lambda)^2 + \omega^4 R_E{}^2 \cos^2\lambda \sin^2\lambda}}\right)$$

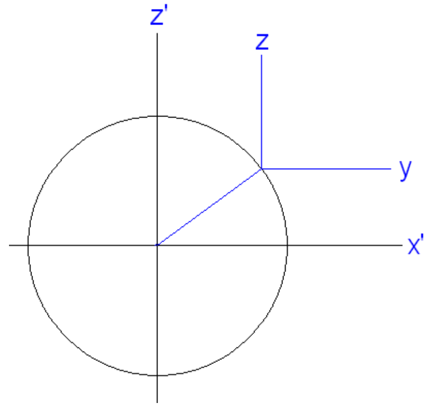
To simplify the expression take $s = \frac{\omega^2 R_E}{g_{\circ}}$, so:

$$\alpha = \cos^{-1}\left(\frac{1 - s\cos^2\lambda}{\sqrt{(1 - s\cos^2\lambda)^2 + s^2\cos^2\lambda\sin^2\lambda}}\right)$$

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{1 + \left(\frac{s\cos\lambda\sin\lambda}{1 - s\cos^2\lambda}\right)^2}}\right) \text{… exact}$$

$$\alpha = \frac{s\sin 2\lambda}{2} \text{… approximate (the angle will be in radians)}$$

Alternatively:



$$\vec{g} = -g_\circ \hat{e}_r + \omega^2 R_E \cos\lambda \hat{y} = -g_\circ \cos\lambda \hat{y} - g_\circ \sin\lambda \hat{z} + \omega^2 R_E \cos\lambda \hat{y}$$

$$|\vec{g}| = \sqrt{(-g_\circ \cos\lambda + \omega^2 R_E \cos\lambda)^2 + g_\circ{}^2 \sin^2\lambda}$$

Change of variable: $s = \dfrac{\omega^2 R_E}{g_\circ}$:

$$|\vec{g}| = g_\circ \sqrt{(-\cos\lambda + s\cos\lambda)^2 + \sin^2\lambda} = g_\circ \sqrt{1 - 2s\cos^2\lambda + s^2\cos^2\lambda}$$

Also:

$|\vec{g}_o| = 9.8$ and:

$$\vec{g} \cdot \vec{g}_o = g_\circ{}^2 - g_o \omega^2 R_E \cos^2\lambda$$

So the angle is:

$$\alpha = \cos^{-1}\left(\frac{\vec{g} \cdot \vec{g}_o}{|\vec{g}||\vec{g}_o|}\right) = \cos^{-1}\left(\frac{g_\circ{}^2 - g_o \omega^2 R_E \cos^2\lambda}{g_\circ{}^2 \sqrt{1 - 2s\cos^2\lambda + s^2\cos^2\lambda}}\right)$$

$$\alpha = \cos^{-1}\left(\frac{1 - s\cos^2\lambda}{\sqrt{1 - 2s\cos^2\lambda + s^2\cos^2\lambda}}\right)$$